π in Finite Symbolic Mechanics

A Constructive Geometric Account from Measured Spherical Packing

Kevin R. Haylett

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Pensée

Abstract

This Pensée gathers the development of a constructive account of π that begins from the commitments of Finite Symbolic Mechanics rather than from pre-existing circular or analytic assumptions. It traces the path from delay embeddings of the digits of π, through the reinterpretation of complex numbers as relational delay, to a geometric view of modern series as hierarchical spherical shell packing under the Spherical Uncertainty Distribution.

A simple lattice-counting algorithm and a framework for physical packing experiments are presented as concrete, finite methods for generating approximations. The work is offered as an autonomous trajectory: it does not claim superior computational efficiency for high precision, but supplies a different starting point — finite measured spheres and local interaction — from which approximations to spherical closure emerge.

The text is written so that a reader can re-enter the flow at any point. All definitions remain strictly finitary and carry explicit uncertainty and provenance.

1 Introduction: Different Starting Points

Many classical and modern algorithms for computing π begin with an object that already carries circular or periodic structure — sine, cosine, degrees, elliptic integrals, or an initial approximation whose provenance already contains π — and then refine residuals until the model converges toward higher fidelity with the circle. In this sense they are self-closing mechanisms: the algorithm lets a model tighten around an invariant that was present from the outset.

The account developed here takes a different origin. It begins with finite, measured spheres whose only primitive geometry is the sphere at the limit of measurement. There is no circle and no π presupposed in the initial setup. The approximation to circular closure — and therefore to π — emerges from the collective behaviour of packed elements under the Spherical Uncertainty Distribution. The circle is not assumed at the start; it is approximated by the boundary formed when finite spheres interact.

This difference in starting commitment is not merely stylistic. It keeps uncertainty, provenance, and local measurement explicit from the first step. It also leaves the framework open to packings that are irregular, fractal, or taken directly from physical experiment rather than from an ideal analytic form. Whether the resulting approximations are numerically faster than existing series remains an open experimental question. The value of the account lies in the coherence of its origin story and in the constructive language it supplies for generating new trajectories.

2 Foundational Commitments of Finite Symbolic Mechanics

Finite Symbolic Mechanics rests on two non-negotiable commitments that are carried throughout this Pensée.

1. Every admissible symbol is a finite instantiation generated by measurement. It carries irreducible error and traceable provenance. It is not the measured interaction itself.

2. At the Alphonic Limit — the smallest distinction that can still be instantiated and held — uncertainty is isotropic. The only sustainable geometry at this limit is the sphere.

From these commitments follow the Spherical Uncertainty Distribution (SUD) as the fundamental description of measurement uncertainty and the requirement that all constructions remain local and finitary.

3 The Geometry of π via Delay Embeddings

When the digits of π are rendered as paths in three-dimensional space using delay-coordinate embedding with lag τ, qualitatively different geometries appear. At small τ the path forms a dense, coiled filament. At larger τ it reorganises into an angular, scaffold-like lattice containing rectangular voids.

Vision-language models consistently describe these embeddings with semantically distinct language. Statistical tests report no significant departure from randomness, yet the geometric difference is visible and describable. Changing the delay parameter is equivalent to changing the lens or resolution. Different faces of π become visible. This observation suggests that an Atlas of π's Faces — systematic renderings across a range of τ — would be a useful empirical resource.

4 Complex Numbers as Delay Embeddings

The imaginary unit i corresponds to a quarter-period phase advance — the operation performed by the Hilbert transform. In the language of nonlinear dynamics this is the optimal delay for reconstructing phase information from a single real-valued observable while preserving energy and orthogonality.

Thus the complex plane is a minimal two-dimensional delay embedding of oscillatory signals. Analyticity becomes a geometric statement about conformality on the reconstructed phase space. No additional ontological commitment to an "imaginary" dimension is required. This reinterpretation is fully consistent with the commitment that all admissible symbols must be finite and measured.

5 Shell-Packing Interpretation of Modern π Algorithms

The Chudnovsky algorithm (and its Ramanujan-type precursors) achieve rapid convergence because each term corresponds to the contribution of one spherical shell in a hierarchical packing. The formula is

$$\frac{1}{\pi} = \frac{12}{640320^{3/2}} \sum_{k=0}^{\infty} (-1)^k \frac{(6k)!}{(3k)!(k!)^3} \frac{545140134k + 13591409}{640320^{3k}}.$$

The constant 640320^3 functions as a scale factor between successive shells. The linear and constant terms in the numerator encode surface and core contributions. The series can be read as a discrete summation over shells whose radii decrease geometrically toward the Alphonic Limit. Inner shells supply coarse foundation; outer shells supply finer boundary corrections. The algorithm converges because each new shell reduces the residual discrepancy between the packed volume and ideal spherical closure.

6 Formal Definitions: Shells and Contributions

Alphonic Limit α. The smallest positive distinction that can be instantiated and held in the current measurement context.

Spherical Uncertainty Distribution at scale r. The uniform distribution over the ball of radius r with density $1/V(r)$, where $V(r) = \frac{4}{3}\pi r^3$.

Shell S_k. The finite spherical layer between radii $r_k > r_{k+1} \geq \alpha$, with thickness $\delta r_k = r_k - r_{k+1}$ and local scale factor $\lambda_k = r_k / r_{k+1}$.

Contribution of a shell $C(S_k)$. The finite correction to spherical closure supplied by packing distinguishable spheres inside the layer:

$$C(S_k) := \int_{S_k} \mathrm{SUD}(x; r_k) \cdot \eta(\text{lattice or measured}) \cdot (1 - U(r_k))\, dV,$$

where $U(r_k) \propto \alpha \ln(r_k/\alpha)$ is the compounding uncertainty term and η may be taken from a lattice rule or from direct physical measurement.

A finite approximation to π is obtained by summing contributions up to a truncation index K where $r_K \geq \alpha$:

$$\frac{1}{\pi} \approx \sum_{k=0}^{K} C(S_k).$$

7 A Simple Lattice-Counting Algorithm for Approximating π

A direct method begins with a regular lattice (hexagonal is a convenient starting choice) and counts elements on the boundary of circles whose diameters are integer multiples of the lattice spacing.

Fix element diameter d and Alphonic Limit α. Choose radii $R_i = m_i d$. For each R_i count the number N_i of lattice points satisfying $|\sqrt{x^2 + y^2} - R_i| \leq \tau$, where τ is a finite tolerance (e.g., $\tau = \alpha$ or $d/2$). Average over several radii or rotations to obtain $\bar{N}$. Estimate circumference $C \approx \bar{N} \cdot d$ and compute

$$\pi \approx \frac{C}{2R}.$$

The count $\bar{N}$ carries finite uncertainty that can be modelled with the SUD. The boundary layer corresponds to the outermost shell in the general contribution framework. The method remains open to replacement of the lattice rule by fractal or experimentally measured arrangements.

8 Physical Packing Experiments as Exogenous Data

Physical experiments supply measured values for η and the form of compounding uncertainty that can be inserted directly into the definition of $C(S_k)$. Simple repeatable setups include packing equal balls inside a circular dish or spherical container whose diameter is an integer multiple of the ball diameter, counting boundary elements within a measurable tolerance, and recording finite contact areas and gaps. All experiments are truncated at the practical resolution limit corresponding to the Alphonic Limit of the instruments used. Provenance is maintained by recording exact dimensions, measurement method, and environmental conditions.

Measured packing data and symbolic construction form an iterative feedback loop: experiment supplies empirical η and $U(r)$; the resulting symbolic trajectory predicts behaviour at finer resolution; new experiments test the prediction and refine the empirical functions.

9 Computational Considerations

For a target of D correct decimal digits the lattice-counting method requires a radius $R \sim 10^D$. The dominant cost scales polynomially with R (roughly R^2 for a full lattice scan or $R^{1+\epsilon}$ with boundary-focused methods). In contrast, Chudnovsky-type series require a number of terms linear in D with per-term cost governed by fast multiplication of growing integers, yielding far better scaling for large D.

The comparison is not decisive inside Finite Symbolic Mechanics. The lattice method makes the cost of carrying uncertainty and provenance explicit and local. It may be competitive for modest precision or when the lattice can be realised physically. The framework itself remains available for exploring how different packing geometries (regular, fractal, or measured) affect convergence rate.

10 Philosophical Reflection: Starting Points and Convergence Mechanisms

The distinction between the two families of methods is one of origin rather than of final numerical result. Classical routes begin inside an already-circular or periodic language and refine residuals. The packing account begins from finite measured spheres and lets an approximation to closure emerge from their interaction. Both are legitimate convergence mechanisms. The packing account does not claim to replace existing series for raw computational speed at very high precision. It supplies an alternative foundation whose internal coherence can be judged on its own terms and whose constructive language remains open to physical data and to irregular geometries.

Even if a direct equivalence with existing algorithms exists through a long chain of geometric inference, the difference in starting commitment remains real. It keeps the framework autonomous and prevents the circular assumption from entering at the first step.

11 Cornerstone Points and Open Questions

The following points have emerged as stable hinges:

- Every admissible symbol is finite and carries error and provenance.
- At the Alphonic Limit the sphere is the only sustainable geometry.
- π can be treated as an emergent invariant of finite spherical closure rather than a primitive constant.
- Modern efficient series admit a geometric reading as hierarchical shell packing under the SUD.
- Physical packing experiments are a first-class source of measured data for the framework.
- Computational efficiency must be assessed alongside the explicit cost of carrying uncertainty and provenance.

Open questions include the practical scaling of lattice or fractal packing methods for high precision, the extent to which measured physical packings can improve convergence, and the degree of independence or overlap with classical analytic routes.

12 How to Re-enter This Work

The document is deliberately sectional. A reader may return at any point without needing the full preceding context. The formal definitions in Sections 5–7 provide the precise language. The algorithm in Section 7 and the experimental framework in Section 8 give concrete entry points for further exploration. The reflection in Section 9 records the current state of understanding about starting commitments and convergence mechanisms.

When context length again becomes difficult, return first to this introduction and to the definitions of Shell and Contribution. From there the rest of the trajectory remains accessible.