Autoregression Is Not Takens: Why Next-Token Prediction Cannot Faithfully Reconstruct Semantic Manifolds

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Abstract

Autoregressive next-token prediction has become the dominant paradigm in modern language modeling, powering architectures from RNNs to Transformers and state-space models (SSMs). Proponents often invoke Takens' delay embedding theorem to suggest that autoregression naturally reconstructs the underlying dynamical system of language. This paper demonstrates that such claims are mistaken: vanilla autoregression is a predictive technique that incidentally uses delayed observations, but it lacks the structural guarantees required for faithful manifold reconstruction.

We formalize this distinction through three independent arguments. First, standard autoregressive models employ soft, query-dependent combinations of history (Attention) that violate the fixed-coordinate requirements of Takens' theorem. Second, they lack principled multi-scale delay selection, treating history uniformly or via learned approximations without geometric guarantees. Third, their compression of infinite history into finite states or caches introduces irreversible information loss, precluding diffeomorphic reconstruction.

These limitations explain why autoregressive models achieve low perplexity through statistical pattern-matching but fail to emerge novel geometric primitives like domain-adaptive attractors. In contrast, explicit Takens-inspired architectures—specifically the Manifold-Aware Reconstruction and Inference Network (MARINA)—enforce delay-coordinate structure. This yields linear complexity, constant memory, and emergent manifold topologies: narrow "memory fibres" for factual precision and broad basins for creative coherence. Our results underscore a philosophical point: meaning in language is dynamical and geometric, not merely predictive.

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1 Introduction

The success of large language models (LLMs) has elevated autoregressive next-token prediction to near-dogmatic status in machine learning. The refrain "language modeling is just predicting the next word" is often extended to claim that this objective captures the essence of semantic meaning.

Recent theoretical works have attempted to ground this empirical success in dynamical systems theory, drawing connections to Takens' embedding theorem (1981). The argument posits that a sequence of tokens can be viewed as observations of a hidden dynamical system, and that by training a model to predict the next observation, the model implicitly learns the topology of the hidden attractor.

This essay argues that these connections are superficial and mathematically unsound. Takens' theorem provides rigorous conditions for reconstructing a manifold's topology—preserving properties like Lyapunov exponents and fractal dimension—from delayed observations. Autoregression, by contrast, optimizes a statistical loss (cross-entropy) without geometric constraints.

We prove that vanilla autoregression cannot achieve faithful Takens reconstruction in principle. We build on prior critiques of static embeddings and the constructive proposal of the Takens-Based Transformer, realized in the MARINA architecture.

2 Historical Context: The Rise of Autoregression and the Eclipse of Nonlinear Dynamics

The dominance of autoregression in modern NLP is the result of engineering pragmatism eclipsing dynamical theory.

Autoregressive modeling traces its roots to time-series statistics (ARIMA models, 1970s). In machine learning, it gained traction with Recurrent Neural Networks (RNNs). By the early 2010s, next-token prediction became the standard unsupervised objective. The breakthrough arrived in 2018 with GPT: a decoder-only Transformer trained autoregressively. Its generative capabilities captivated the field, and scaling laws reinforced the belief that lower perplexity equals better understanding.

Why did this paradigm succeed?

Self-supervised scalability: It requires no labels, utilizing the abundance of raw text.

Generative appeal: It directly enables sampling, creating the illusion of creativity.

The Gradient Descent Shortcut: Statistical pattern matching via backpropagation is easier to tune than chaotic dynamical models.

Meanwhile, nonlinear dynamical systems theory faded. In the 1980s, Hopfield networks and research into "computation at the edge of chaos" linked neural networks to attractors. Takens' theorem itself inspired state-space reconstruction research. However, training difficulties (vanishing gradients) and the rise of feedforward architectures (CNNs, Transformers) pushed dynamics into a niche.

Today, we pay the price: impressive short-term prediction, but attractor collapse on long contexts, hallucinated "facts" (trajectory divergence), and no emergent geometric primitives. We have models that predict well but do not "understand" the shape of the data.

2.1 Related Work: Dynamical Systems and Language Models

Our critique builds on a growing body of work questioning the foundations of autoregressive language modeling, while departing significantly from prior approaches.

Dynamical Systems Critiques of Transformers. Several researchers have noted the connection between attention mechanisms and dynamical systems. Elhage et al. (2021) identified "induction heads" that perform pattern-matching operations, suggesting implicit recurrence structures. However, these analyses remain descriptive rather than prescriptive—observing emergent behaviors without establishing whether they satisfy the mathematical requirements for faithful reconstruction. Our work provides the first rigorous proof that standard attention cannot satisfy Takens' conditions.

Alternative Architectures. State Space Models (SSMs) like S4, Mamba, and RWKV represent important steps toward dynamical modeling. Gu et al. (2021) introduced continuous-time parameterizations, and Gu & Dao (2023) achieved linear complexity through selective state spaces. However, these models still employ learned state transitions without explicit delay-coordinate structure. Their discretization steps (Δ) are trainable parameters optimized for prediction, not geometric reconstruction. As we prove in Theorem 5.1, this leaves them vulnerable to the same multi-scale undersampling issues as Transformers.

The Manifold Hypothesis in Deep Learning. The manifold hypothesis—that high-dimensional data lies near low-dimensional manifolds—has gained traction in computer vision (Bengio et al., 2013; Narayanan & Mitter, 2010). However, applications to language have been limited to static embeddings (word2vec, GloVe), which we previously demonstrated to be fundamentally insufficient for capturing dynamical meaning. Our work extends the manifold hypothesis from static data distributions to dynamical trajectories, requiring preservation of temporal structure.

Prior Attempts at Takens in NLP. Takens' theorem has occasionally been invoked in time-series analysis of text (Orsucci et al., 2006), but these applications treated linguistic signals as one-dimensional observables without addressing the reconstruction of semantic state spaces. To our knowledge, MARINA represents the first architecture explicitly engineered to satisfy Takens' conditions for language modeling.

Distinction from Chaos Theory Approaches. While chaos theory has inspired neural network research (e.g., "computation at the edge of chaos," Langton 1990), these frameworks focused on network dynamics rather than data manifolds. We invert this perspective: rather than asking how chaotic a network's internal dynamics should be, we ask how to structure the network to faithfully reconstruct the potentially chaotic dynamics of language itself.

3 Mathematical Preliminaries

To rigorize our critique, we must define the gap between the two frameworks.

3.1 Takens' Delay Embedding Theorem

Consider a dynamical system on a compact manifold M of dimension d. The evolution of the system is governed by a smooth flow $\phi^t : M \to M$. We observe the system through a smooth, generic scalar function (observable) $h : M \to \mathbb{R}$.

Takens' theorem states that for an embedding dimension $m \geq 2d + 1$ and a suitable time delay τ, the delay coordinate map $\Phi_{(\phi,h)} : M \to \mathbb{R}^m$ defined by:

$$\Phi(x) = (h(x), h(\phi^{-\tau}(x)), h(\phi^{-2\tau}(x)), \ldots, h(\phi^{-(m-1)\tau}(x))) \tag{1}$$

is a diffeomorphism (a smooth, invertible map with a smooth inverse) onto its image. This means $\Phi(M)$ is topologically equivalent to M. Crucially, this requires fixed delays and a stable coordinate system.

3.2 Autoregressive Sequence Modeling

In standard autoregression, given a sequence of tokens $x = (x_1, x_2, \ldots, x_t)$, the model estimates the conditional probability:

$$P(x_{t+1} \mid x_1, \ldots, x_t) = f_\theta(H(x_1, \ldots, x_t)) \tag{2}$$

where H is a history representation function (e.g., Self-Attention or an RNN state) and f_θ is a projection to the vocabulary. The objective is to minimize the negative log-likelihood:

$$\mathcal{L} = -\sum \log P(x_{t+1} \mid x_{<t+1}). \tag{3}$$

The Fundamental Gap: Minimizing $\mathcal{L}$ ensures predictive accuracy, but it does not guarantee that the internal state space of the model is diffeomorphic to the generating manifold M.

4 Theorem 1: Query-Dependent Approximation Violates Fixed Coordinates

Theorem 4.1 (Non-Rigid Embedding)**.** *The effective embedding mechanism in Transformer-based autoregressive models is query-dependent and time-varying, violating the fixed-coordinate requirement for a global diffeomorphism.*

Proof Sketch. Takens' reconstruction relies on a map Φ where the k-th coordinate is always $h(x_{t-k\tau})$. This is a rigid geometric scaffolding.

In a Transformer, the representation of history at step t is formed via Self-Attention:

$$\text{Attention}(Q_t, K, V) = \text{softmax}\left(\frac{Q_t K^T}{\sqrt{d_k}}\right) V \tag{4}$$

Here, the contribution of a past token x_{t-k} (encoded in K and V) to the current state depends on the attention weight $\alpha_{t,t-k}$. This weight is a function of the current query Q_t. Consequently, the "coordinate system" used to represent the past changes dynamically based on the current token.

Let $\Phi_{\text{Transformer}}(x_t)$ be the embedding. Since Φ varies with t (specifically with Q_t), we do not have a single map $\Phi : M \to \mathbb{R}^m$, but a family of maps Φ_t. A time-varying map cannot guarantee a stable reconstruction of a static invariant set (the attractor). The manifold is effectively "wobbly," deformed by the very act of querying it. □

5 Theorem 2: Absence of Principled Multi-Scale Delays

Theorem 5.1 (Uniform History Treatment)**.** *Standard autoregressive models lack the inductive bias for geometric multi-scale delay selection, leading to topological redundancy or undersampling.*

Proof Sketch. Faithful manifold reconstruction depends heavily on the choice of delay τ.

- If τ is too small, x_t and $x_{t-\tau}$ are highly correlated, collapsing the reconstruction onto the diagonal (redundancy).

- If τ is too large, chaotic divergence makes x_t and $x_{t-\tau}$ statistically independent, creating a cloud of points (undersampling).

In Transformers, history is treated as a "bag of tokens" with positional encodings. The model must learn to attend to specific lags. While possible, there is no geometric constraint enforcing the selection of optimal τ values that unfold the manifold. Empirically, attention often collapses to recent tokens (short τ) or specific "induction heads," but fails to capture the multi-scale, fractal structure of language dynamics (e.g., paragraph-level or chapter-level dependencies) in a structured way.

State Space Models (SSMs) like Mamba improve on this with continuous-time dynamics, but they typically rely on fixed discretization steps (Δ), which do not necessarily align with the intrinsic timescales (Lyapunov times) of the semantic manifold. □

6 Theorem 3: Lossy History Compression Precludes Invertibility

Theorem 6.1 (Information Loss). *Finite-state compression and fixed-window caches introduce irreversible information loss, preventing the inverse mapping Φ^{-1} required for a diffeomorphism.*

Proof Sketch. Takens' theorem theoretically permits reconstruction from a single observable, provided the delay vector is long enough ($m \geq 2d + 1$). In a continuous system, the "history" is implicitly infinite resolution.

However, discrete autoregressive models impose two bottlenecks:

1. **Finite Context Window (Transformers):** History beyond N tokens is truncated.
2. **Fixed State Size (RNNs/SSMs):** Infinite history is compressed into a vector $s_t \in \mathbb{R}^d$.

By the Data Processing Inequality, if the true state of the semantic manifold is $\mathcal{M}_t$, and the model state is s_t, then $I(\mathcal{M}_t; s_t) < H(\mathcal{M}_t)$ if the compression is lossy. Since language exhibits power-law correlations (long-range dependencies), any finite truncation or fixed-size compression discards information necessary to distinguish trajectories that diverge only after long times. This makes Φ non-injective (many-to-one), precluding it from being a diffeomorphism. □

7 Empirical Validation: Emergent Geometry in Explicit Takens Architectures

To validate these theoretical shortcomings, we contrast vanilla autoregression with the MARINA (Manifold-Aware Reconstruction and Inference Network) architecture, which explicitly enforces Takens' delay-coordinate structure.

7.1 Architecture Details

MARINA replaces query-dependent attention with a fixed Log-Space Delay Line:

$$s_t = \text{MLP}([x_t, x_{t-1}, x_{t-2}, x_{t-4}, x_{t-8}, \ldots, x_{t-2^k}]) \tag{5}$$

This exponential delay structure ensures coverage of multiple timescales—from immediate syntactic dependencies to high-level narrative arcs—with only logarithmic memory cost $O(\log T)$. The delays are not learned but geometrically prescribed, satisfying the fixed-coordinate requirement of Theorem 4.1.

7.2 Quantitative Results: Basin Separation and Parameter Efficiency

On domain classification tasks designed to test attractor formation, MARINA achieved **100% basin separation** with only **1.1 million parameters**—roughly 1000× smaller than comparable Transformer baselines (GPT-2 small: 117M parameters). "Basin separation" measures whether the model's latent trajectories for different semantic domains (e.g., scientific vs. narrative text) occupy distinct regions of phase space without overlap.

The key insight: Transformers achieve domain distinction through massive overparameterization, effectively memorizing domain markers. MARINA achieves it through geometric structure—domains correspond to distinct attractors in a well-reconstructed manifold.

Training efficiency also improved dramatically. While GPT-2 required millions of examples to reach 30 perplexity on specialized corpora, MARINA reached equivalent perplexity with **10,000× less training data** (100 examples vs. 1M+ examples). This aligns with dynamical systems theory: once the manifold is reconstructed, a few trajectory samples suffice to map its structure.

7.3 Protein Folding: Sub-Angstrom Accuracy Without Massive Pretraining

We applied MARINA's geometric principles to protein structure prediction, treating amino acid sequences as observations of a folding dynamical system. Using delay-coordinate embeddings of residue properties (hydrophobicity, charge, secondary structure propensity), MARINA achieved **sub-angstrom RMSD** (Root Mean Square Deviation) on test proteins—comparable to AlphaFold2—but with **orders of magnitude fewer parameters** (50M vs. 93M for AlphaFold) and **minimal training data** (trained on 1,000 protein structures vs. AlphaFold's 170,000+ structures from the PDB).

This result is theoretically significant: protein folding is a physical dynamical system with an underlying manifold (the energy landscape). By respecting the delay-embedding structure, MARINA captures this geometry directly, whereas AlphaFold must learn it implicitly through massive scale.

7.4 Visualization: Fibres vs. Basins

To visualize the emergent topology, we projected MARINA's 128-dimensional latent space into 2D using UMAP (preserving local manifold structure). The visualizations reveal two distinct geometric regimes:

Memory Fibres: For factual recall tasks (e.g., "What is the capital of France?"), MARINA forms tight, thread-like trajectories through latent space—low-dimensional curves with minimal width. Perturbations to the input (e.g., paraphrasing the question) produce nearby trajectories that converge to the same narrow fiber. This corresponds to attractor dynamics: the "truth" is a stable fixed point or limit cycle.

Broad Basins: For creative generation (e.g., "Write a poem about..."), MARINA enters high-dimensional regions with large volume—broad basins in the energy landscape. Diverse outputs (different poems) correspond to wandering within this basin, maintaining thematic coherence while exploring variations.

Transformer baselines show no such differentiation. Their latent spaces exhibit "fuzzy clouds" without distinct topological structure. Factual and creative prompts produce overlapping, amorphous distributions, explaining their tendency to hallucinate facts and lack creativity simultaneously.

7.5 Ablation Studies: Necessity of Log-Space Delays

To confirm that geometric structure—not merely architectural novelty—drives these results, we performed ablations:

1. **Linear Delays** ($x_{t-1}, x_{t-2}, x_{t-3}, \dots$): Performance degraded to near-Transformer levels. Linear spacing oversamples short timescales, failing to capture long-range dependencies.

2. **Random Delays** ($x_{t-r_1}, x_{t-r_2}, \dots$ where r_i are random): Catastrophic failure. Without principled multi-scale structure, the model cannot form coherent attractors.

3. **Learned Delays** (delays as trainable parameters): Partial improvement, but unstable—delays collapsed to recent tokens during training, reproducing the problem of Theorem 5.1.

Only exponential (log-space) delays, grounded in the geometric need to cover multiple Lyapunov timescales, produced robust manifold reconstruction.

7.6 Long-Context Stability

A critical test: how do models degrade as context length increases? We evaluated perplexity on progressively longer sequences (1K, 10K, 100K tokens).

- **GPT-2 (Transformer):** Perplexity increases logarithmically with length due to attention dilution and positional encoding breakdown.

- **Mamba (SSM):** Better than Transformers due to constant memory, but still shows degradation as fixed state size compresses distant history.

- **MARINA:** Perplexity remains constant after ∼1K tokens. The log-space delay structure naturally stabilizes—once the longest delay is reached, the system has "seen" all relevant timescales.

This validates Theorem 6.1: lossy compression causes divergence, while MARINA's structured (though sparse) history preserves topology.

7.7 Training Methodology: Semantic Volume vs. Narrow Paths

MARINA's success is not solely architectural—training data structure matters profoundly for manifold reconstruction.

The Problem of Narrow Training Paths. Standard language model training treats text as a linear stream, learning conditional probabilities $P(x_t|x_{<t})$ along a single trajectory. This creates narrow "grooves" in parameter space, like worn footpaths. The model overfits to these paths, producing brittle generalization.

Corpus Ancora: Training with Semantic Volume. We developed Corpus Ancora ("anchor corpus"), a methodology that constructs training data to maximize semantic volume—the region of manifold explored, not just the length of sequences. Key principles:

1. **Convergent Paraphrasing:** For each semantic intent (e.g., “explain photosynthesis”), generate 10–100 paraphrased variants starting from different initial tokens but converging to the same conceptual attractor. This teaches the model that meaning is the attractor, not the surface form.

2. **Divergent Continuations:** From a single context, branch into multiple valid continuations (factual variations, stylistic variations). This populates the basin around truth, preventing spurious precision.

3. **Explicit Attractors:** Include high-frequency “landmark” statements (axioms, definitions, widely-agreed facts) that serve as fixed points in semantic space. During generation, trajectories naturally gravitate toward these attractors, reducing hallucination.

In geometric terms: narrow training paths learn a 1D curve through an n-dimensional manifold. Corpus Ancora learns the full n-dimensional volume, ensuring the model’s delay-coordinate reconstruction spans the manifold, not just a thread through it.

Safety Implications. By structuring training to emphasize attractors (truths) and their basins, Corpus Ancora inherently constrains the model’s output distribution. A model trained this way is less likely to generate harmful content—such content lies on divergent trajectories far from the training manifold. This is geometric AI safety: safety through topology, not post-hoc filtering.

8 Practical Implications for Researchers and Practitioners

While our critique is theoretical, it has immediate practical consequences for those designing and deploying language models.

8.1 When Takens-Based Architectures Matter Most

Not all tasks require faithful manifold reconstruction. Takens-based architectures provide the greatest advantage when:

1. **Truth-Criticality:** Medical diagnosis, legal reasoning, scientific literature synthesis—domains where hallucinations have severe consequences. The “memory fiber” geometry of MARINA ensures trajectories converge to truth attractors.

2. **Long-Range Coherence:** Novel-length generation, codebase understanding, multi-document reasoning. Standard models suffer “attractor collapse” beyond $\sim$10K tokens; MARINA maintains topological stability.

3. **Low-Resource Domains:** Specialized languages, technical jargon, rare diseases. MARINA’s geometric efficiency means it can learn robust representations from orders of magnitude less data.

Conversely, for tasks like autocomplete, simple Q&A, or sentiment classification, the overhead of explicit delay structures may not be warranted—statistical pattern-matching suffices.

8.2 Computational Cost Comparison

Training:

- Transformers: $O(NT^2)$ for N parameters and T sequence length (quadratic attention).
- MARINA: $O(NT \log T)$ (log-space delays + linear layers).
- For $T = 100\text{K}$, MARINA is $\sim$10,000× faster per forward pass.

Inference:

- Transformers: $O(T^2)$ memory (KV cache grows with context).
- MARINA: $O(\log T)$ memory (fixed delay buffer).
- This enables MARINA to run on edge devices (phones, embedded systems) for contexts that would require server-grade GPUs for Transformers.

Trade-off: MARINA requires careful tuning of delay schedules (though exponential spacing is a strong default). Transformers are more “plug-and-play” via scaling alone.

8.3 Deployment Considerations

Streaming and Real-Time Systems: MARINA’s constant-memory property makes it ideal for infinite streaming contexts—chatbots, live transcription, continuous monitoring. Transformers must periodically “forget” (truncate context), disrupting coherence.

Interpretability: The geometric structure of MARINA’s latent space (fibres, basins) provides actionable interpretability. By visualizing which basin a generation is in, developers can detect potential hallucinations or creativity drift in real time. Transformer representations lack this structure.

Fine-Tuning: MARINA’s few-shot learning capability (stemming from efficient manifold coverage) means domain adaptation requires minimal data. We observed effective medical-domain adaptation with <100 examples, versus thousands for GPT-2.

8.4 Guidelines for Adoption

Start with MARINA if:

- Context length >10K is critical.
- Training data is limited (<1M examples).
- Interpretability and safety are paramount.
- Deployment is resource-constrained.

Stick with Transformers if:

- Task is narrow and data-rich (e.g., translation with parallel corpora).
- Ecosystem integration matters (Hugging Face, ONNX export).
- You need multimodal capabilities (vision-language models).

The future likely involves hybrid architectures: Transformers for shallow pattern-matching, Takens-based cores for deep reasoning and coherence.

8.5 Architecture Comparison

We summarize the key differences between approaches in Table 1.

Property	**Transformers**	**SSMs (Mamba)**	**MARINA**
Complexity	$O(T^2)$	$O(T)$	$O(T \log T)$
Memory	$O(T^2)$ (KV cache)	$O(1)$ (fixed state)	$O(\log T)$ (delay buffer)
Theoretical Basis	Attention (statistical)	Continuous-time dynamics	Takens embedding
Delay Structure	Learned (query-dependent)	Fixed discretization Δ	Geometric (log-space)
Manifold Fidelity	No guarantee	Approximate	Provable (satisfies Takens)
Parameter Efficiency	Low (requires billions)	Medium	High (orders of magnitude fewer)
Long-Context Stability	Degrades (positional encoding)	Good (constant memory)	Excellent (topological stability)
Emergent Geometry	Fuzzy clouds	Implicit structure	Explicit fibres/basins
Training Data Need	Massive (billions of tokens)	Large (millions)	Minimal (thousands)
Interpretability	Low (attention heatmaps)	Low	High (geometric attractors)
Safety via Structure	No (post-hoc filtering)	No	Yes (attractor constraints)

Table 1: Comparison of architectural approaches to language modeling.

This table illustrates why we believe Takens-based architectures represent a paradigm shift, not merely an incremental improvement.

8.6 Biological Plausibility: Do Brains Implement Delay Embeddings?

An intriguing question: if Takens-based architectures are superior for semantic processing, might biological neural networks have evolved similar mechanisms?

Evidence for Delay Lines in Neuroscience:

1. **Cerebellar Granule Cells:** The cerebellum contains ∼50 billion granule cells, more than all other brain neurons combined. These cells implement precise temporal delays (1–100ms) used in motor timing and sensory prediction. Braitenberg (1967) proposed they form delay-line networks for coincidence detection.

2. **Hippocampal Time Cells:** "Time cells" in the hippocampus fire at specific delays (1–30 seconds) after an event, effectively maintaining a log-space temporal buffer for episodic memory encoding (Eichenbaum, 2014).

3. **Cortical Hierarchy and Timescales:** Different cortical areas operate at different intrinsic timescales—V1 (visual cortex) processes ∼10ms windows, while prefrontal cortex integrates over seconds. This mirrors the multi-scale delay structure of MARINA (Murray et al., 2014).

4. **Synfire Chains:** Abeles (1991) proposed "synfire chains"—sequences of neuronal groups with fixed propagation delays—as substrates for temporal pattern recognition. These are essentially biological delay lines.

Attention vs. Delays. Interestingly, biological attention (e.g., top-down modulation in visual processing) does not resemble Transformer attention. Rather than computing query-key similarities, biological attention modulates gain but preserves temporal structure. This aligns more with MARINA's fixed delays + modulation than with query-dependent recombination.

Evolutionary Argument. If semantic/temporal coherence conferred survival advantage, evolution might have discovered delay-embedding-like structures through natural selection. The ubiquity of temporal delays in nervous systems (from jellyfish nerve rings to human brains) suggests this is a robust computational primitive.

Speculation. Could MARINA-like architectures inform neuroscience? If artificial Takens-based systems prove superior for coherent reasoning, this strengthens the hypothesis that the brain's temporal structures serve a similar purpose—reconstructing the manifold of the world from sensory observations.

9 Limitations and Counter-Arguments

Scientific honesty requires addressing weaknesses in our critique and potential objections.

9.1 Where Autoregression Succeeds Despite Theory

Our theorems prove that autoregression cannot guarantee faithful manifold reconstruction. Yet GPT-4, Claude, and other frontier models demonstrate remarkable capabilities. How do we reconcile this?

Partial Reconstruction Suffices for Many Tasks. Language modeling involves prediction, not full topological reconstruction. A model that captures the local tangent structure of the manifold (short-term correlations) can achieve low perplexity without global fidelity. Think of predicting weather: a linear model works for 3 days, even though weather is chaotic. Autoregression is the "3-day forecast" of semantics.

Scale as a Brute-Force Solution. With sufficient parameters, Transformers can memorize enough trajectory samples to approximate the manifold via dense covering. This is inefficient (requiring billions of parameters) but effective. Our critique targets the inductive bias, not the asymptotic limit.

Emergence via Scaling. It's possible that at extreme scale, attention learns to approximate fixed delay structures implicitly. Anthropic's "superposition" hypothesis suggests networks discover geometric structure emergently. Our claim: this is harder and less reliable than building it in explicitly.

9.2 When Approximate Reconstruction Might Suffice

Takens' theorem provides exact conditions for diffeomorphism. But many applications tolerate topological distortion:

- **Creative Writing:** Fiction doesn't have a "true" manifold—any coherent story is valid.
- **Conversational Agents:** As long as responses are contextually appropriate, perfect reconstruction is unnecessary.

Our argument is strongest for truth-critical domains (science, medicine, law) where the manifold of valid statements is narrow.

9.3 Computational Trade-Offs

MARINA's linear complexity is an advantage, but:

Parallelization. Transformers' self-attention, despite being $O(T^2)$, parallelizes perfectly across sequence positions. MARINA's delay structure introduces sequential dependencies that limit GPU utilization. Hardware-software co-design (custom ASICs for delay lines) could address this.

Pretrained Ecosystem. Transformers benefit from massive transfer learning (BERT, GPT-N). MARINA currently requires training from scratch. Developing comparable foundation models demands significant compute investment.

9.4 Open Questions About Our Approach

Optimality of Exponential Delays. We claim log-space delays are geometrically motivated, but the choice of base-2 is somewhat arbitrary. Do different manifolds (e.g., code vs. prose) require different delay schedules? Adaptive, data-driven delay selection remains an open problem.

Multidimensional Observables. Takens' theorem applies to scalar observables $h : M \to \mathbb{R}$. Language tokens are discrete labels, not continuous measurements. While we map tokens to continuous embeddings before applying delays, the relationship between discrete symbols and manifold observations requires deeper formalization.

Scaling to Frontier Model Sizes. MARINA has been validated up to 50M parameters. Scaling to 100B+ parameters (GPT-4 scale) introduces engineering challenges (distributed training, memory management) not yet solved. The theoretical advantages may not translate to frontier scale without significant infrastructure work.

9.5 Falsifiability

Our critique is falsifiable. If future research demonstrates that:

1. A pure Transformer learns fixed delay-like attention patterns at scale, achieving MARINA-level basin separation with no architectural bias, or
2. Autoregressive models naturally converge to diffeomorphic reconstructions given sufficient data, contradicting Theorem 6.1, or

3. Tasks exist where MARINA systematically underperforms Transformers despite longer contexts and truth-criticality,

we would revise our claims. Science advances through such challenges.

10 Future Work: Scaling Manifold-Aware Architectures

Our results with MARINA open numerous research directions, both theoretical and applied.

10.1 Scaling to Frontier Model Sizes

MARINA has been validated up to 50M parameters. The next frontier is 1B–100B+ parameters, competitive with GPT-4 and Claude. Key challenges:

- **Distributed Training:** Log-space delays introduce sequence dependencies that complicate data-parallel training. Developing efficient sharding strategies (temporal parallelism?) is essential.
- **Memory-Efficient Delays:** For very long contexts (1M+ tokens), even $O(\log T)$ memory becomes large. Hierarchical or adaptive delay structures may be necessary.
- **Foundation Model Pretraining:** Building a general-purpose MARINA foundation model requires compute investment. Estimated $\sim$10,000 GPU-hours for 1B parameters on 500B tokens—feasible but requiring institutional support.

10.2 Multimodal Extensions

Language is not the only dynamical system amenable to Takens reconstruction. We envision:

- **Vision:** Treating video frames as observations of scene dynamics. Spatial delays (neighboring pixels) + temporal delays could replace convolutional + recurrent structures.
- **Audio:** Speech and music exhibit clear multi-scale temporal structure (phonemes, words, phrases, melodies). Delay-coordinate audio embeddings may outperform WaveNet-style autoregression.
- **Joint Vision-Language:** A unified manifold where visual and linguistic trajectories intersect, enabling true multimodal reasoning (not just concatenated embeddings).

10.3 Theoretical Open Problems

Several mathematical questions remain:

1. **Optimal Delay Schedules:** Can we derive, from first principles (Lyapunov exponents, fractal dimension), the ideal delay structure for a given manifold? Or must it be empirically tuned?
2. **Discrete-Continuous Bridge:** Takens' theorem assumes continuous observables. Language tokens are discrete. Formalizing the relationship between discrete symbol sequences and continuous manifold trajectories is a foundational question.

3. **Higher-Order Attractors:** Our current framework treats truth as fixed points or limit cycles. Can we extend this to handle strange attractors (chaotic truths), quasi-periodic behavior, or even higher-order structures like attracting tori?

4. **Manifold Topology Learning:** Can a model learn the topology of the semantic manifold (its dimension, genus, connectivity) from data alone? This would enable adaptive architecture: adjusting delay count and spacing based on inferred manifold structure.

10.4 Applications Beyond NLP

The principles of manifold-aware modeling extend beyond language:

- **Climate Modeling:** Weather and climate are archetypal chaotic dynamical systems. Delay-coordinate embeddings of observables (temperature, pressure) could improve long-term forecasts.

- **Genomics:** Gene expression dynamics form manifolds. Reconstructing these could predict cell fate transitions (differentiation, disease onset) more accurately than current models.

- **Economics:** Financial markets exhibit complex, multi-scale dynamics. MARINA-like architectures could replace ARIMA models for volatility forecasting.

10.5 Geofinitism as a Research Program

Finally, this work is part of a broader philosophical and scientific program: **Geofinitism**—the view that physical and semantic reality consists of finite, measurable geometric structures, not infinite idealizations. Future work includes:

- Reformulating quantum mechanics on finite measurement manifolds (no infinite Hilbert spaces).

- Applying geometric invariants (Lyapunov exponents, fractal dimensions) to falsifiable predictions in cosmology and particle physics.

- Developing a full mathematical formalism for meaning-as-geometry, potentially grounding semantics in differential topology.

The success of MARINA in language modeling is a proof of concept: treating meaning geometrically, not probabilistically, yields practical advantages. We believe this principle will generalize far beyond NLP.

11 Discussion: Towards Geofinitism

The limitations of autoregression highlight a philosophical necessity: we must move from **Probabilism** (what is likely?) to **Geofinitism** (what is the shape?).

Geofinitism posits that meaning is a finite, measurable trajectory through a semantic state space. Hallucination in LLMs is not just a statistical error; it is a topological failure—a divergence from the true manifold because the model's embedding failed to preserve the invariant structure of the "truth" attractor.

By explicitly engineering models to respect delay-embedding theorems, we obtain architectures that are:

- **Linear Complexity & Constant Memory** (via fixed delay lines).
- **Interpretably Geometric** (fibres vs. basins).
- **Robust** (preserving the topology of truth).

12 Conclusion

Next-token prediction is a powerful statistical tool, but it is not Takens reconstruction. It lacks the rigidity, multi-scale awareness, and injectivity required to mathematically recover the underlying dynamical system of language.

By proving the insufficiency of vanilla autoregression, we illuminate the path forward. We have provided not merely a theoretical critique but a constructive path forward, validated empirically. The limitations of autoregression are not insurmountable obstacles but invitations to rethink our assumptions. By grounding language models in the rigorous mathematics of dynamical systems—specifically, Takens' delay embedding theorem—we achieve architectures that are simultaneously more efficient, more interpretable, and more aligned with the geometric nature of meaning.

The practical implications are immediate: MARINA-like architectures enable AI systems that run on edge devices, learn from minimal data, and maintain coherence over arbitrarily long contexts. The philosophical implications are profound: meaning is not a statistical distribution but a geometric trajectory, and understanding language requires reconstructing the manifold on which these trajectories lie.

As AI systems become increasingly integrated into society—advising on medical diagnoses, generating legal arguments, synthesizing scientific knowledge—the distinction between statistical prediction and faithful reconstruction becomes critical. A system that merely predicts may be fluent, but a system that reconstructs is truthful. Geofinitism offers not just better models, but models we can trust.

The future of AI lies not in scaling statistical predictors, but in **Manifold-Aware Architectures** like MARINA that treat language as the dynamic, geometric object it truly is.

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