Finite Symbolic Algebra

Memory-Mapped States, Dynamic Semantic Lattices, and the Endogenous Transformation of Compressed Trajectories

A measurement-first foundation for the preparation, positioning, transformation, and reinjection of finite symbols

No symbol operates from nowhere. No flow is available without retained difference. No algebraic transformation stands outside the symbolic region it changes.

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This monograph records a stage of continuing research within Finite Symbolic Mechanics. Its provisional notation is intended to preserve and extend the present conceptual trajectory rather than to declare a closed algebraic calculus.

Abstract

This monograph develops the preliminary foundation of Finite Symbolic Algebra as a regional continuation of Finite Symbolic Mechanics, Finite Symbolic Dynamics, and Finite Symbolic Logic. Its starting point is that conventional mathematics and FSM alike manipulate compressed symbolic trajectories. A written term does not operate as a bare mark. It must be recognised, classified, reconstructed, prepared, and finitely positioned within a local symbolic region before it can participate in a transformation. The assumptions supporting the term are not merely metadata attached to an otherwise complete object; they constitute the functional trajectory through which the term becomes operable.

The argument begins at the measurement boundary. An exogenous interactional continuation is inferred, while the symbolic system has direct access only to finite registrations. Registrations are admitted into symbolic classes and retained through finite memory mappings. Flow is then reconstructed endogenously as the measurably ordered comparison of present and memory-mapped discrete states. The discreteness belongs to the registered states and does not by itself establish that the inferred exogenous field is ontologically discrete. Memory does not restore the past occurrence; it produces a present provenance-bearing state admitted as standing for an earlier registration.

The monograph introduces the Dynamic Semantic Lattice as a finite, irregular, memory-bearing, and changing relational organisation of unique symbolic occurrences. Each node has finite extent, provenance, uncertainty, scale, and a locally measured relational position. A Nodal Value Function evaluates and updates what a node can presently do within a regional container. Prepared symbols are injected or reinjected into that region, altering both available continuations and the reconstruction of earlier states. The resulting Functional Symbolic Trajectory is the registered succession of these changing configurations. Semantic flow therefore names a measured order of relational transformations rather than an independently transported substance called meaning.

Finite Symbolic Logic supplies the governing constraints: unique occurrence, Alphonic distinction, registered ordering, provenance-bearing transition, propagating uncertainty, scale locality, endogenous reasoning, compression without identity, and declared or reconstructed admissibility. Finite Symbolic Algebra inherits those constraints and studies a specialised cycle of selection, reconstruction, preparation, positioning, transformation, coherence measurement, recompression, and reinjection. Equality becomes an admissible permission to substitute one compressed trajectory for another within a declared locality, rather than literal identity between occurrences or their formative histories.

The purpose of the present work is not to complete an algebra. It is to establish a coherent conceptual and provisional formal container from which an algebra can be developed without

returning silently to ideal repeatable symbols, unmeasured order, context-free equality, or a reasoner positioned outside the symbolic flow.

Prefatory Note

This work arose from an attempt to ask what algebra becomes when the finite production and ordering of its symbols are restored. The first response was to attach extent, uncertainty, provenance, scale, and admissibility to algebraic terms. That response was useful but incomplete because it still treated a symbol as a compact object to which contextual information could be added. The more consequential recognition was that the symbol is already a compression of the Functional Symbolic Trajectories required to make it operable.

That recognition brought algebra back into the path established by Finite Symbolic Logic. The logical arrow had already been reconstructed as a compression of realised transitions, and reasoning had been defined as the endogenous measurement, comparison, ordering, and continuation of finite symbolic flow. Algebra could not then be founded upon objects and operations placed outside that flow. It had to be located as a specialised form of endogenous reasoning within a finite symbolic region.

The discussion subsequently widened. A symbol must be prepared prior to injection or reinjection into the region in which it is to operate. Its finite position is relational rather than an ideal coordinate. The surrounding region is a Dynamic Semantic Lattice whose nodes are unique occurrences and whose relations change with registration, memory, weighting, and continuation. The lattice belongs to a wider Alphon and corpus of language, because mathematical notation is held by the words, symbols, learned rules, and reconstruction procedures through which it becomes usable.

The resulting picture is necessarily recursive. This monograph is written in the same words-within-words field that it describes. Its terms compress wider trajectories and ask the reader to reconstruct them through another finite history. The aim is therefore not total decompression. Such an attempt would extend the contextual container every time it used another word. The aim is sufficient recoverability: to unfold enough of the supporting trajectory that the location, conditions, and direction of the present work remain visible.

Reader's Orientation

The argument proceeds through six movements. Part I begins at the measurement boundary and distinguishes inferred exogenous continuation from finite endogenous registration. It then places memory at the origin of available flow. Part II locates mathematical language within the wider Alphon and explains why words-within-words requires regional compression. Part III develops the Dynamic Semantic Lattice, real finite positioning, the Nodal Value Function, and the reinjection cycle. Part IV carries the principles of Finite Symbolic Logic into Finite Symbolic Algebra and examines equality, equivalence, algebraic law, measured numbers, and a simple transformation. Part V situates basins, attractors, separatrices, saddle points, manifolds, and neural-network correspondences within the model. Part VI gathers the trajectory into a provisional schema and research programme.

The formulas are compressions of the accompanying prose. They are not authorities placed above it. The notation remains provisional because no single tuple or arrow can contain the full provenance of a realised symbol. Its purpose is diagnostic: to make visible where a conventional expression relies upon a measurement bridge, memory relation, classification, regional preparation, or compressed transition that would otherwise remain undeclared.

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Part I

Measurement, Registration, and Reconstructed Flow

Chapter 1

Why Algebra Must Begin Before the Operation

1.1 The inherited starting point

Conventional algebra ordinarily begins after its terms have been prepared. Numbers, variables, operators, equality, domains, and rules of substitution are already available. Once these have been declared or learned, the algebraist can ask whether an operation is closed, associative, commutative, distributive, or invertible. The visible calculation begins after a larger symbolic history has been compressed into the notation.

An early FSM response might enrich each term with a tuple containing its value, uncertainty, provenance, scale, and admissibility. That approach retains useful information, but it still permits the term to appear as a completed entity carrying supplementary attributes. The deeper starting point is that the term itself functions only because a finite trajectory can be reconstructed around it. The term is not first an object and later a bearer of history. It is a compression whose operability depends upon that history.

This moves the foundational question from "What objects and operations form the algebra?" to "What finite symbolic sequence must be reconstructed for this occurrence to perform its present function?" The question applies equally to a numeral, variable, operator, equality sign, bracket, diagram, or verbal instruction. Every one of them must be distinguished, classified, ordered, and related within a finite field before an operation can proceed.

1.2 The proposed location of algebra

Finite Symbolic Algebra is not introduced as a separate foundation placed beside Finite Symbolic Logic. Logic already describes the finite ordering of symbols into an admissible flow, while reasoning describes the endogenous measurement and continuation of that flow. Algebra is a specialised regional practice within this framework. It selects compressed mathematical trajectories, reconstructs enough of their conditions for a local operation, transforms them, measures the coherence of the proposed continuation, and recompresses the result.

The provisional definition is therefore:

> *Finite Symbolic Algebra is the endogenous preparation, positioning, transformation, coherence measurement, recompression, and reinjection of finite symbolic trajectories within a locally admissible mathematical region.*

This definition deliberately places preparation before operation and reinjection after recompression. An algebraic result does not leave the field as a completed object. It returns as another finite occurrence, changes the available context, and becomes part of the provenance from which the next operation proceeds.

Chapter 2

The Exogenous Boundary

2.1 Interaction before symbolic state

FSM begins with interaction rather than an independently completed symbolic world. A local measurement procedure may be represented provisionally as

$$\mathcal{W}^{\text{exo}} \dashrightarrow M_L \longrightarrow r_t, \tag{2.1}$$

where $\mathcal{W}^{\text{exo}}$ names an inferred exogenous interactional field, M_L is a measurement interaction local to a finite system, and r_t is a distinguishable registration. The dashed arrow prevents the notation from implying direct access to the exogenous field. The field is inferred as a condition of registrations; it is not encountered already divided, named, or ordered in the terms used by the symbolic system.

This distinction guards against an easy return to conventional nouns. Signals, currents, chemical interactions, particles, waves, and forces may provide powerful models, but their names belong to later symbolic trajectories. At the boundary, the system obtains a finite registration. It does not obtain the exogenous continuation together with its final ontology.

2.2 Discrete registration without exogenous atomism

Separate symbolic states must be distinguishable. If r_i and r_j are admitted as two registrations under a comparison procedure D_L, their difference must meet the operative Alphonic boundary:

$$D_L(r_i, r_j) \geq \alpha_L. \tag{2.2}$$

This gives endogenous discreteness. It does not establish that the exogenous field is composed of the same discrete states. The measurement interaction, threshold, sampling, calibration, encoding, and available medium participate in producing the distinction. FSM therefore maps the inferred continuation into discrete registrations without promoting that mapping into a completed picture of what exists beyond it.

This is an important restraint upon the later dynamical language. When the monograph speaks of lattice states or nodal transitions, it is speaking first about registered symbolic states. The inferred exogenous conditions may be described through those states, but the description remains a reconstruction across the measurement boundary.

Chapter 3

From Registration to Symbol

3.1 Admission into a symbolic class

A registration becomes a symbol when it is admitted into a rule-governed or socially stabilised field of repeatable distinction. An encoding or classification operation can be written as

$$E_L(r_t) \longrightarrow \sigma_t. \tag{3.1}$$

The occurrence σ_t has finite extent, scale, provenance, uncertainty, and relational placement. It does not become an instance of an unextended ideal sign merely because it has been classified. The classification allows the system to continue as though selected differences were irrelevant for a particular purpose.

Two occurrences may therefore be related by

$$\sigma_i \sim_A \sigma_j, \tag{3.2}$$

where A denotes a symbolic class and $\sim_A$ is an admissible equivalence. The relation does not erase the occurrence difference. If there were no difference, there would not be two registrations to compare.

3.2 Occurrence before type

The priority of occurrence over type will later become decisive for algebra. A repeated letter a does not return the same occurrence. It produces a_i, a_j, and perhaps a_k, which may be admitted under the same type A. Conventional notation suppresses the indices because the equivalence is stable across the intended operation. FSM retains the suppression as a useful compression while refusing to promote it into literal identity.

This distinction also prevents a semantic lattice from becoming a dictionary of ideal words. A node is first a finite occurrence. A type-node, if used, is a higher compression formed across an equivalence class of occurrences. The compressed type can be functional without becoming foundationally prior to the registrations from which it was formed.

Chapter 4

Memory-Mapped States

4.1 Memory as a new registration

A present registration alone cannot establish a trajectory. Some finite relation to an earlier state must remain available. We represent the mapping of a symbolic occurrence into a retained state as

$$\sigma_t \longrightarrow \mu_t, \tag{4.1}$$

where μ_t is a memory-mapped state. The medium may be a changed neural configuration, a computer memory state, a written mark, a recording, a learned weight, or another finite carrier. The general requirement is not permanence but sufficient retention for later comparison.

Memory does not preserve the earlier occurrence itself. At a later stage, what is available is a present reconstruction or retained alteration admitted as provenance-related to the past occurrence:

$$\widehat{\mu}_{t-1} \neq \mu_{t-1}, \qquad \widehat{\mu}_{t-1} \sim_H \mu_{t-1}. \tag{4.2}$$

The hat records that the provenance may be partial, compressed, transformed, or reconstructed. A person does not compare the present directly with a past event that has returned intact. The person compares present registrations, one of which is positioned as a memory of an earlier event. The same applies to an instrument log or computational state: retention is another finite implementation, not an escape from finite occurrence.

4.2 Memory as a condition of order

For a system to register before and after, it must retain enough difference to compare two mapped states. If

$$D_L(\widehat{\mu}_{t-1}, \mu_t) \geq \alpha_L \tag{4.3}$$

and the relevant ordering procedure supports

$$\widehat{\mu}_{t-1} \prec_{L,k} \mu_t, \tag{4.4}$$

then the system can construct an ordered transition. The coordinate k may be temporal, dependency-based, spatial, causal-attributive, evidential, or another declared order. The notation must not allow one coordinate to be silently substituted for another.

Without retention there may be a present registration, but no available relation by which that registration is measured as following another. Memory is therefore constitutive of symbolic flow rather than an optional archive added after the flow has occurred.

Chapter 5

Flow as Reconstructed Order

5.1 The finite trajectory

A finite memory-bearing trajectory can be represented as

$$\Gamma_t = (\widehat{\mu}_{t-m}, \ldots, \widehat{\mu}_{t-1}, \mu_t) \,. \tag{5.1}$$

The sequence does not contain the original past occurrences. It contains currently available mapped states carrying finite provenance and declared order. Flow is therefore the endogenous reconstruction of an ordered difference across retained states.

> *Registration, retained state, measured difference, and registered order together make finite symbolic flow available.*

This formulation neither denies exogenous change nor asserts that a substance called flow has been directly observed. It preserves the boundary: an exogenous continuation is inferred as participating in the production of registrations, while flow is available to the symbolic system through the ordered comparison of those registrations.

5.2 Three uses of flow

The term *flow* can now be separated into three related uses. An inferred exogenous continuation is the unobserved interactional condition reconstructed from measurement. A registered state flow is the finite ordered succession of distinguishable symbolic states. A semantic flow is the relational reconstruction through which those states acquire functional direction and support continuation.

These uses should not be collapsed. The semantic trajectory may be coherent while misdescribing the inferred exogenous conditions. The registered sequence may be accurate while its causal interpretation remains unresolved. Conversely, a useful exogenous model may depend upon a coarse registered flow whose internal distinctions become inadequate at another scale. The bridge among these levels carries provenance and uncertainty.

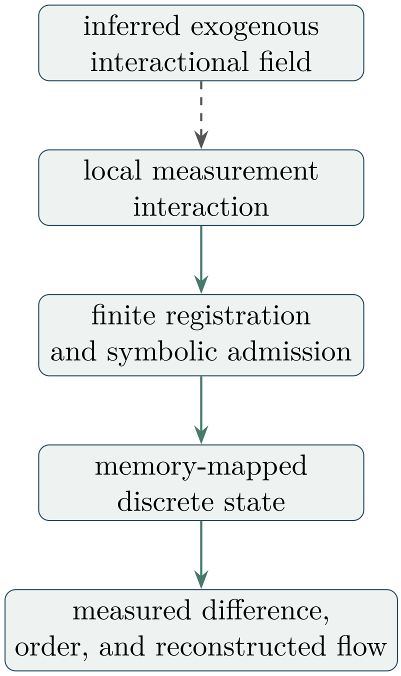

Figure 5.1: The exogenous continuation is inferred across the measurement boundary. Flow becomes symbolically available through finite registration, retention, distinction, and order.

Part II

The Alphon, Corpus, and Regional Compression

Chapter 6

The Wider Alphon

6.1 Available resources of symbolic formation

The Alphon names the wider operational field of distinguishable symbolic resources from which trajectories can be formed. It includes letters and words, but also numerals, operators, diagrams, codes, sounds, gestures, and other admitted marks. It should not be treated as a timeless completed alphabet. The available Alphon is historically, materially, and technically situated.

We may write

$$\mathcal{A}_e = \{\text{symbolic formation resources available in epoch } e\}\,, \tag{6.1}$$

while remembering that the braces are themselves a conventional compression. The operational content is a finite and changing collection of distinguishable resources and formation practices.

The corpus is formed through registered trajectories constructed from those resources:

$$\mathcal{A}_e \longrightarrow \mathcal{C}_e. \tag{6.2}$$

The corpus contains inherited sentences, equations, proofs, diagrams, definitions, memories, institutional conventions, and learned reconstruction procedures. It is open and historically growing. No actual reasoner accesses it as a completed whole.

6.2 Language as the wider container

Mathematics operates within this broader field. Its notation is narrower and more tightly constrained, but its symbols are named, taught, interpreted, verified, and connected to measurement through language. Even an austere formula depends upon a surrounding trajectory that states what its marks are allowed to do.

The relationship can be expressed provisionally as

$$\mathcal{A} \supset \mathcal{C} \supset \mathcal{L}_{\text{math}} \supset \mathcal{R}_{\text{alg}} \supset \Sigma_{\text{local}}, \tag{6.3}$$

provided that the containment signs are read functionally rather than as claims about completed ideal sets. Each narrower region draws upon reconstruction procedures carried by the wider region while suppressing most of them from immediate display.

Chapter 7

Words Within Words

7.1 Recursive symbolic support

A symbol is unfolded through other symbols. The algebraic occurrence a may be explained through *variable*, *domain*, *substitution*, and *value*. Each explanatory term is another compression requiring further words. The trajectory therefore expands recursively:

$$\sigma \rightsquigarrow \Sigma_1 \rightsquigarrow \Sigma_2 \rightsquigarrow \cdots . \tag{7.1}$$

This does not require an actually infinite sequence. Every realised explanation is finite. It means that no finite symbolic explanation reaches a final unextended word standing outside symbolic dependence. The stopping point is local and functional: enough of the trajectory has been reconstructed for the present distinction or operation.

FSM is subject to the same condition. Its account of symbols, measurement, memory, and flow is formed in words and equations. It does not acquire a privileged metalanguage merely because it describes the symbolic boundary. The theory enters the corpus as another trajectory, alters the expectation field of later readers, and can itself be reconstructed, compressed, disputed, and revised.

7.2 The impossibility of total contextual display

If every assumption supporting a simple expression were displayed, the explanation would require further symbols whose assumptions would also need display. The contextual region would expand with the attempt to complete it. This is why mathematical writing does not show the entire language system required to read each line.

The absence of total display is not in itself a failure. Finite communication requires compression. The important distinction lies between recoverable local compression and a compression promoted into unsupported universality. A regional convention may be entirely functional within its tested range while becoming misleading when transferred across scale, discipline, implementation, or measurement regime.

The proper demand is therefore bounded:

> *Unfold enough of the regional container to recover the distinctions, transformations, and bridges upon which the present claim depends.*

This principle keeps the framework usable. It does not ask each equation to carry the whole corpus. It asks that foundational authority not be assigned to a compressed term when the trajectory required to support that authority is unavailable or concealed.

Chapter 8

Functional Symbolic Regional Containers

8.1 The local working field

A functional symbolic regional container is the finite, locally available structure within which a class of symbols and transformations can operate without reconstructing the entire wider corpus. It contains a selected vocabulary, admissible classifications, ordering conventions, memory states, scales, uncertainties, goals, and transformation procedures.

A locality may be represented as

$$L = (W, \alpha, H, \Delta, C, S, G)\,, \tag{8.1}$$

where W is the available window, α the operative Alphonic boundary, H accessible provenance, Δ uncertainty, C admissibility conditions, S scale, and G the current goal or direction. These components remain finite descriptions rather than complete independent quantities.

The regional container is not merely background information. It prepares the terms and determines what operations can be reconstructed. The visible expression

$$x + 1 = 2 \tag{8.2}$$

is an interface into such a container. Its use depends upon recognition of symbol occurrences, a domain for x, meanings for the numerals and operators, a reading order, a permissible transformation, a goal, and a verification procedure. These conditions are normally compressed because the regional decoder has been stabilised through education and repeated use.

8.2 Bridges between regions

A conclusion admitted in one region does not transfer automatically to another. Let

$$\Sigma_{L_1} \vdash_{L_1} q. \tag{8.3}$$

Transfer to L_2 requires a finite bridge B_{12} that establishes which distinctions and relations are preserved:

$$B_{12} : (\Sigma_{L_1}, q) \longrightarrow (\Sigma_{L_2}, q') \,. \tag{8.4}$$

The terminal symbol q' may resemble q while carrying altered scale, uncertainty, provenance, or admissibility. Reusing the same inscription does not establish an unchanged trajectory. This will later matter whenever an algebraic identity is moved from a formal region into measurement.

Part III

The Dynamic Semantic Lattice

Chapter 9

A Finite Relational Field

9.1 Definition of the lattice

The Dynamic Semantic Lattice is the currently active relational organisation of finite symbolic occurrences. A local lattice state may be represented as

$$\mathcal{R}_t = (N_t, W_t, \mathcal{M}_t)\,, \tag{9.1}$$

where N_t is the finite collection of available nodal occurrences, W_t is the finite structure of their measured relations, and $\mathcal{M}_t$ is the available memory mapping of earlier configurations.

The word *lattice* is used as a functional model and not as an importation of a uniform classical grid. The nodes are not identical points, the spacing is not assumed uniform, and the relations are not guaranteed to satisfy a universal metric. Each occurrence has extent, provenance, uncertainty, scale, and a particular relational history.

The word *dynamic* carries two forms of change. New occurrences and relations may be registered, while existing occurrences may also acquire altered functional positions when the surrounding trajectory changes. A final word can redirect the reconstruction of an earlier sentence. A later mathematical declaration can change how an earlier symbol is classified. The physical inscription may remain in place while the functional semantic position changes.

9.2 The lattice as memory-bearing

The lattice cannot be defined only by a present configuration. Without some mapping of earlier configurations it cannot construct a trajectory through its changes. The memory component $\mathcal{M}_t$ therefore participates in every local valuation that depends upon before, after, return, repetition, novelty, expectation, or deviation.

This does not mean that the lattice contains a perfect recording of its entire past. Its memory is finite, selected, transformed, and unevenly accessible. Some paths are strongly stabilised through repetition, while others have been compressed beyond local recovery. The resulting semantic geometry is historical without being completely historical: the present region carries traces sufficient for some reconstructions and not for others.

Chapter 10

Occurrence Nodes and Type Compressions

10.1 Unique nodal occurrence

Let N_i and N_j be two finite nodes. Even when both are admitted under a common symbolic class A, they remain different occurrences:

$$N_i \neq N_j, \qquad N_i \sim_A N_j. \tag{10.1}$$

Their relational weightings are likewise not exactly identical:

$$\mathbf{w}_i \neq \mathbf{w}_j. \tag{10.2}$$

A biological model may call two cells neurons while recording different connections, thresholds, positions, and histories. A computational model may use repeated architectural units while their weights or activations differ. A written page may repeat the glyph a while every occurrence occupies another position and participates in another local sequence.

10.2 Compression into a type-node

For practical use, a region may compress an occurrence class into a type-node:

$$\{N_i, N_j, N_k, \ldots\} \xRightarrow{K_A} N_A. \tag{10.3}$$

The type-node allows the system to reuse a compact symbolic function without displaying every occurrence. It is not identical to the occurrence history from which it was formed. Its stability depends upon a range in which the suppressed differences do not alter the intended operation.

This gives two useful resolutions: an occurrence lattice grounded in unique finite registrations and a compressed type lattice used for efficient symbolic continuation. Conventional mathematics ordinarily begins at the type resolution. FSM retains the occurrence resolution as the measurement condition that makes symbolic repetition possible.

Chapter 11

Real Finite Positioning

11.1 Preparation before placement

A symbol cannot operate as a bare mark. It must be prepared for the region into which it will be injected. We write

$$P_{\mathcal{R},L}\left(\sigma_i \mid H, G, \mathcal{M}\right) \longrightarrow \widetilde{\sigma}_i^{\mathcal{R}}, \tag{11.1}$$

where the tilde indicates a finite occurrence prepared for regional use. Preparation may include recognition, classification, domain assignment, reconstruction of provenance, estimation of uncertainty, activation of relevant memory paths, and selection of permitted transformations.

The same visible mark can therefore acquire different prepared functions. The letter a may operate as a word, a variable, an index, a coefficient, a matrix entry, or a label. Its visual similarity does not supply its complete functional position.

11.2 Position as measured relation

Real finite positioning does not require an independently existing Euclidean semantic coordinate. It requires a finite relational placement that is measurable and consequential within the local region. We may write

$$\mathrm{Pos}_{\mathcal{R}}(\sigma_i) = \left(w_{i1}, w_{i2}, \ldots, w_{in}\right), \tag{11.2}$$

where the w_{ij} record locally available relations to neighbouring occurrences or compressed classes. These relations may concern temporal order, syntactic dependence, substitution, similarity, opposition, provenance, scale, uncertainty, or expected continuation.

The position is real in the FSM sense because it is supported by finite registrations and because changing it alters the trajectory. It is nevertheless model-relative. No claim is made that semantic position exists as a hidden geometrical point independently of the measurement and symbolic procedures that construct it.

Chapter 12

The Nodal Value Function

12.1 A local valuation

The Nodal Value Function performs the local work through which an occurrence acquires an operative value within a region. It does not reveal a fixed semantic content stored inside the node. It measures what the node can presently do from its relational position.

A provisional schema is

$$\mathcal{V}_{i,L}^{(t)} : \left(\sigma_i, \mathcal{N}_i^{(t)}, \mathbf{w}_i^{(t)}, \mathcal{M}_t, H_i, \Delta_i, S_i, G_t\right) \longrightarrow \left(\mathbf{v}_i^{(t+1)}, \mathcal{D}_i^{(t+1)}\right), \tag{12.1}$$

where $\mathcal{N}_i^{(t)}$ is the available relational neighbourhood, $\mathbf{w}_i^{(t)}$ the present weighting, $\mathcal{M}_t$ the accessible memory field, H_i provenance, Δ_i uncertainty, S_i scale, and G_t local direction. The output $\mathbf{v}_i^{(t+1)}$ is a structured local valuation, while $\mathcal{D}_i^{(t+1)}$ is the resulting region of admissible continuations.

The notation does not require a universal scalar value. Relevance, admissibility, uncertainty, trajectory direction, and relational tension may not be reducible to one number without concealing the distinctions the function is intended to retain.

12.2 Value is regional

The value depends upon the region:

$$\mathbf{v}_i^{(t)} = \mathcal{V}_{i,L}^{(t)}\left(\sigma_i \mid \mathcal{R}_t\right). \tag{12.2}$$

If the region changes, the valuation may change:

$$\mathcal{R}_t \neq \mathcal{R}_{t+1} \quad \Longrightarrow \quad \mathbf{v}_i^{(t)} \neq \mathbf{v}_i^{(t+1)}. \tag{12.3}$$

The implication is not an assertion that every regional change must alter every nodal value. It states that identity of value cannot be presumed merely because the visible occurrence remains unchanged. The surrounding path may activate a different reconstruction basin and thereby reposition the node functionally.

Chapter 13

Injection, Revaluation, and Reinjection

13.1 The endogenous cycle

A selected compression is reconstructed, prepared, and returned to the active field:

$$\mathcal{R}_t \xrightarrow{\text{select}} \sigma_i \xrightarrow{\text{reconstruct}} \widehat{\Sigma}_{\sigma_i} \xrightarrow{P_{\mathcal{R},L}} \widetilde{\sigma}_i \xrightarrow{I_{\mathcal{R}}} \mathcal{R}_{t+1}. \tag{13.1}$$

The new region is not formed by simple set addition. The injected occurrence changes the available relationships:

$$\mathcal{R}_{t+1} \neq \mathcal{R}_t \cup \{\sigma_i\}, \qquad W_{t+1} \neq W_t. \tag{13.2}$$

Nodal Value Functions then remeasure the locally affected positions. The resulting changes may be compressed into another symbol or expression and reinjected once more. The operation is endogenous because each output becomes part of the field from which the next valuation proceeds.

13.2 The Functional Symbolic Trajectory

The registered succession of lattice configurations forms a Functional Symbolic Trajectory:

$$\Gamma_{\mathcal{R}} = (\mathcal{R}_0, \mathcal{R}_1, \ldots, \mathcal{R}_n)\,. \tag{13.3}$$

The word *functional* indicates that the trajectory carries operative relations and consequences rather than merely displaying a chronological list. The word *symbolic* identifies the level of registered distinction. The word *trajectory* indicates an ordered, provenance-bearing passage through changing states.

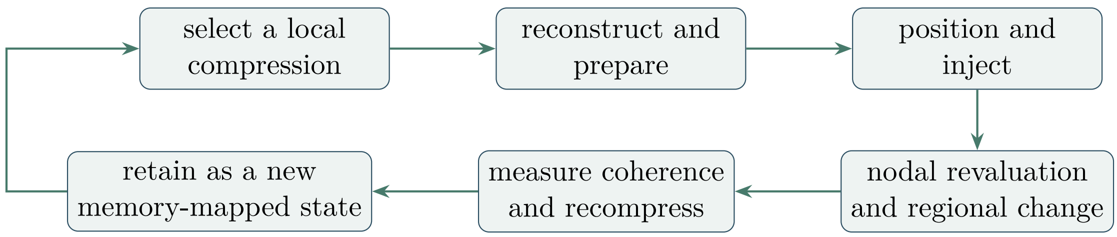

Figure 13.1: The endogenous symbolic cycle. Recompression does not leave the process; it becomes retained state and re-enters the available region.

Part IV

From Finite Symbolic Logic to Finite Symbolic Algebra

Chapter 14

Endogenous Measured Coherence

14.1 Coherence as a registered relation

Coherence is not a substance possessed by an expression, nor a judgement delivered by an observer outside the symbolic field. It is a measured relation between a proposed continuation and the finite trajectory presently available within a locality.

We may write

$$M_L^{\mathrm{coh}}\left(\Gamma_{[t-m,t]}, T, \sigma_{t+1}\right) \longrightarrow c_{t+1}, \tag{14.1}$$

where T is a proposed transformation and c_{t+1} is a finite registration of admission, rejection, or unresolved coherence. The judgement enters the field and changes the conditions of later judgement. It is therefore endogenous.

Endogenous does not mean self-certifying. The available history may be incomplete, an uncertainty may be understated, a type classification may fail, or an ordering coordinate may be silently changed. A later trajectory can remeasure the earlier one under another locality. There is recursion without a final external reasoner.

14.2 A vector of local coherence

Coherence should initially be treated as a structured local description rather than forced into one universal scalar. A provisional vector is

$$\mathbf{C}_L(T) = (C_d, C_o, C_p, C_s, C_\delta, C_a), \tag{14.2}$$

where the components concern distinction continuity, ordering continuity, provenance continuity, scale compatibility, uncertainty propagation, and operational admissibility. A transformation is admitted when its measured description lies within the local admissible region:

$$\mathbf{C}_L(T) \in \mathcal{D}_L. \tag{14.3}$$

The boundary of $\mathcal{D}_L$ is itself a finite symbolic construction. Different fields may employ different tests, tolerances, and forms of proof. The common FSM requirement is that the conditions of admission remain declared or recoverable when they become consequential.

Chapter 15

The Inheritance from Finite Symbolic Logic

15.1 Logic and reasoning

Finite Symbolic Logic defines logic as the finite ordering of symbols into an admissible flow. It defines reasoning as the locally situated measurement, comparison, reordering, and adaptive continuation of that flow. Finite Symbolic Algebra is therefore a particular reasoning practice operating in a strongly stabilised mathematical region.

The logical arrow already compresses a finite history of transformations. Algebraic operators and equality signs inherit the same condition. Their compact inscriptions do not cease to carry provenance merely because the supporting path has been omitted from the page.

15.2 The ten inherited constraints

The ten working principles of Finite Symbolic Logic carry directly into algebra. Table 15.1 records the correspondence without treating the principles as a closed axiomatic system.

Table 15.1: Finite Symbolic Logic principles inherited by Finite Symbolic Algebra.

Principle	**Algebraic consequence**
Finite registration	Every variable, numeral, operator, bracket, equality sign, and result is realised through a finite occurrence.
Unique occurrence	Repeated appearances of a term are distinguishable registrations admitted under a common symbolic class.
Alphonic distinction	Algebraic marks and their ordering must remain separately registerable under the operative medium and scale.
Registered order	The order of symbols and the order of transformations belong to the trajectory rather than to an external index alone.
Provenance-bearing transition	Every rewrite arrow compresses a realised or reconstructable transformation history.

Principle	**Algebraic consequence**
Propagating uncertainty	Uncertainty may enter through classification, reconstruction, ordering, transformation, compression, and comparison.
Scale locality	An algebraic equivalence holds within a declared symbolic and operational region; transfer requires a bridge.
Endogenous reasoning	Performing, checking, or explaining an algebraic manipulation produces further finite states in the field.
Compression without identity	A shortened expression is not identical to the trajectory or expression from which it was constructed.
Declared or reconstructed admissibility	Domains, substitution rules, identities, and verification criteria must be available where they condition a claim.

These constraints do not abolish conventional algebra. They identify the conditions under which its compressions remain stable and the places at which those conditions may become visible.

Chapter 16

The Algebraic Term as Compressed Trajectory

16.1 Reconstruction of a single term

The mark a is finite, but it cannot act algebraically in isolation. Within a local region it may activate a trajectory that distinguishes the mark, classifies it as a letter, admits it as a variable, assigns a domain, connects repeated occurrences, permits specified substitutions, and relates the term to neighbouring operators.

We represent the bounded reconstruction as

$$D_L\,(a \mid \Sigma_W, H, G) \longrightarrow \widehat{\Sigma}_a. \tag{16.1}$$

The reconstruction is local and purpose-relative. The same mark unfolds differently in $a+b$, $a \in A$, $\forall a$, and a_{ij}. The surrounding occurrences constrain the trajectory required to make the mark operational.

The trajectory is not assumed to be fully stored inside the symbol. The mark acts as a coupling point into supporting paths distributed across the expression, prior definitions, learned conventions, memory, and the wider regional corpus.

16.2 Decompression is not exact recovery

The term *decompression* can mislead if it suggests that one complete original trajectory was packed into the mark and can later be restored without difference. In general,

$$D_L(K(\Gamma)) \longrightarrow \widehat{\Gamma}', \qquad \widehat{\Gamma}' \sim_L \Gamma, \tag{16.2}$$

rather than $D_L(K(\Gamma)) = \Gamma$. The reader or machine constructs a trajectory sufficiently coherent for the present operation. Another locality may construct a different path from the same visible compression.

This is not a removable accident of communication. It follows from finite memory, unique occurrence, regional positioning, and words-within-words. Mathematical training stabilises the reconstruction so strongly that it can appear automatic, but the shared decoder remains part of the operative event.

Chapter 17

Equality, Substitution, and Trajectory Equivalence

17.1 Equality as permission

Two expressions placed on opposite sides of an equality sign are materially and sequentially different occurrences. Their construction histories may also differ. Equality cannot therefore mean literal identity of inscription or trajectory.

Within Finite Symbolic Algebra, a local relation

$$K_1 =_L K_2 \tag{17.1}$$

can be read as permission to substitute the trajectory reconstructed from K_1 for that reconstructed from K_2 without producing a relevant distinguishable change in the declared operation.

More explicitly,

$$D_L(K_1) \sim_{\mathcal{O},L} D_L(K_2), \tag{17.2}$$

where $\sim_{\mathcal{O},L}$ denotes operational equivalence under $\mathcal{O}$ in locality L. The relation may be exact within a formal regional container while remaining a measured classification across different finite occurrences.

17.2 Several forms of sameness

The framework benefits from distinguishing symbolic identity, type equivalence, operational substitutability, and provenance relation. Provisional signs may include

$$\begin{aligned}
\sigma_i &=_{\mathrm{s}} \sigma_j && \text{for identity of a selected inscriptional encoding,} && (17.3)\\
\sigma_i &\sim_A \sigma_j && \text{for admission under a common symbolic class,} && (17.4)\\
K_1 &\sim_{\mathcal{O},L} K_2 && \text{for operational substitutability,} && (17.5)\\
\widehat{\mu}_{t-1} &\sim_H \mu_{t-1} && \text{for a provenance-bearing memory relation.} && (17.6)
\end{aligned}$$

The notation remains open to revision. Its purpose is to prevent a single equality sign from silently carrying several different judgements.

Chapter 18

Algebraic Laws as Regional Coherences

18.1 Associativity and ordering

In an ideal algebraic structure, associativity is written

$$(a + b) + c = a + (b + c). \tag{18.1}$$

In a realised finite operation, the two sides describe different transformation orders. Rounding, truncation, saturation, finite precision, uncertainty propagation, or scale change may make those paths distinguishable. The FSM relation is therefore closer to

$$(A \oplus B) \oplus C \sim_{\delta,L} A \oplus (B \oplus C), \tag{18.2}$$

where the two trajectories are admitted as indistinguishable within a declared region and distinction tolerance. Classical associativity appears as the trajectory-insensitive limit in which the suppressed operational differences remain irrelevant.

18.2 Identity, inversion, and lost distinction

The identity relation $a + 0 = a$ depends upon what the occurrence 0 has been prepared to mean. It may represent no counted event, a reading below resolution, an instrument baseline, a balanced difference, or a stipulated origin. These trajectories can share an inscription without being interchangeable in every region.

Inversion exposes compression more sharply. The formal relation

$$(a + b) - b = a \tag{18.3}$$

does not guarantee recovery of the earlier finite state if the intermediate operation removed distinctions. Symbolic reversal may return the same visible numeral, numerical reversal may return an indistinguishable value, and trajectory reversal would require reconstruction of the earlier provenance-bearing state. The third may be unavailable:

$$T^{-1}(T(A, B), B) \not\equiv_H A. \tag{18.4}$$

Algebra cannot recreate distinctions that have passed below the available boundary merely by writing an inverse sign. It can construct a symbol consistent with what remains.

Chapter 19

A Worked Elementary Transformation

19.1 Distinct occurrences of the same type

Consider

$$a + a = 2a. \tag{19.1}$$

The three visible occurrences of a are not one repeated physical event. We may expose them as

$$a_i + a_j = 2a_k, \qquad a_i \sim_A a_j \sim_A a_k. \tag{19.2}$$

Before addition occurs, the system has already registered three occurrences, classified them under a shared type, and admitted coupled substitution. Conventional algebra begins after these equivalence judgements have been compressed.

19.2 Different trajectories, coherent result

The left side constructs an addition trajectory,

$$\Gamma_+ = T_+(a_i, a_j), \tag{19.3}$$

while the right side constructs a coefficient or multiplication trajectory,

$$\Gamma_\times = T_\times(2, a_k). \tag{19.4}$$

The trajectories are not identical:

$$\Gamma_+ \neq \Gamma_\times. \tag{19.5}$$

They may nevertheless be measured as operationally coherent within the algebraic region:

$$\Gamma_+ \sim_{\mathrm{alg},L} \Gamma_\times. \tag{19.6}$$

The expression $2a_k$ is a shorter compression admitted in place of a_i+a_j. The transformation suppresses the distinction between two repeated additions and one coefficient operation because that distinction is locally irrelevant to the intended continuation. If the provenance of the two occurrences mattered, as it might for two independent measurements rather than one measurement reused, the same compression could become inadmissible.

19.3 The complete regional cycle

The elementary manipulation can now be represented as a cycle. The occurrences are selected and reconstructed; their type equivalence is measured; the addition trajectory is formed; a coefficient compression is prepared; operational coherence is assessed; and the new expression is reinjected into the proof or calculation. The result then changes the available window from which the next step proceeds.

This is why algebraic manipulation is endogenous. The final line is not a timeless endpoint. It is a new memory-mapped symbolic state carrying the transformation history, even when that history is no longer displayed.

Chapter 20

Counted, Measured, and Set Numbers

20.1 Different formations beneath one numeral

The same numeral can be formed through different trajectories. A counted number arises through the registration of distinct events. A measured number arises through comparison, instrument response, calibration, resolution, and encoding. A set number is stipulated within a formal or metrological regional container.

We may provisionally mark these formations as

$$n^{[c]}, \qquad x^{[m]}, \qquad k^{[s]}. \tag{20.1}$$

The notation is not intended for every ordinary calculation. It exposes an epistemically consequential difference that conventional notation often suppresses.

20.2 Propagation through an expression

A set number does not acquire empirical uncertainty merely because it appears beside a measured value, and its symbolic exactness does not remove the uncertainty of the measurement through which it is realised. A counted value may carry uncertainty in classification or missed registration even when its final numeral is an integer. A measured value carries the finite conditions of the comparison that produced it.

Finite Symbolic Algebra will require typed transformation rules that preserve these formation differences where they affect the result. This provides a direct continuation into the wider FSM work on measured numbers, constants, and the removal of uncertainty through definition. The algebraic expression may be exact within its set relations while the bridge to and from measurement remains finite and uncertain.

Part V

Dynamical Geometry and Computational Correspondence

Chapter 21

Basins of Reconstruction

21.1 Basin and attractor

A basin is a finite symbolic region in which different perturbations tend to produce related reconstructions or continuations. Once a term is prepared inside elementary algebra, the expression $x + 1 = 2$ tends to activate subtraction, balance, and the continuation $x = 1$. The basin is constructed through learned rules, repeated examples, notation, goals, and institutional reproduction.

An attractor is a recurrently stabilised pattern of continuation. It should not be reified as an external object pulling symbols through a smooth space. It is a measured regularity across finite trajectories. Repeated returns strengthen the expectation field and make one continuation more readily reconstructed than neighbouring alternatives.

21.2 Separatrix and saddle point

A separatrix is a finite relational boundary at which a small change in preparation, scale, or context sends the trajectory into another basin. The expression $x^2 = 1$ has different admissible continuations when x is positioned among integers, real numbers, complex numbers, matrices, or other structures. The visible expression alone does not contain the boundary conditions that direct its path.

A saddle point is a symbolic position stable under some perturbations and unstable under others. Terms such as *energy*, *information*, *dimension*, *measurement*, and *proof* can be highly stable within a local discipline while branching sharply when transferred across a regional boundary. Their apparent solidity is produced by local repetition rather than universal identity of reconstruction.

Chapter 22

Finite Reconstructed Manifolds

22.1 A model of relational geometry

The word *manifold* can describe the locally traversable geometry reconstructed from the Dynamic Semantic Lattice, provided that it is not silently imported as a smooth completed continuum. An FSM manifold is formed from finite nodes, measured relations, memory, and admissible transitions.

Its curvature can be used to describe changing continuation. A relatively flat region admits similar transitions across neighbouring paths. A sharply curved region changes the available continuation rapidly when the local context is perturbed. The geometry is functional: it records relations among symbolic trajectories rather than asserting that meaning occupies an independently existing higher-dimensional space.

22.2 Finite dynamics rather than a picture of things

Basins, attractors, separatrices, saddle points, and manifolds are therefore modelling terms within Finite Symbolic Dynamics. They organise measured regularities across symbolic state changes. They are not nouns that complete the world by naming hidden semantic objects.

This preserves the process orientation of FSM. The useful question is not “Where is the meaning-object located?” but “Which finite relational changes make this continuation available, stable, or divergent?” The semantic landscape is reconstructed from trajectories and changes with the trajectories that traverse it.

Chapter 23

Correspondence with Neural-Network Technology

23.1 A productive mapping

Neural networks provide one finite technical region in which aspects of the model can be implemented and measured. Nodes or hidden-state locations provide finite computational positions; parameters provide relational constraints; activations provide changing states; recurrent or contextual mechanisms provide memory; and ordered computation produces a trajectory through configurations.

The correspondence is summarised in Table 23.1.

FSM formulation	Approximate computational correspondence
Finite node occurrence	Neuron, unit, token occurrence, memory address, or hidden-state location
Relational weighting	Synaptic relation, parameter, attention weight, or graph edge state
Nodal Value Function	A wider functional analogue of activation, aggregation, gating, and contextual evaluation
Dynamic Semantic Lattice	Changing computational graph or hidden-state relational geometry
Regional container	Active layer, context window, recurrent state, or selected subgraph
Prepared symbol	Encoded, projected, or contextually reconstructed input state
Reinjection	Recurrent update, residual return, memory update, or next-token context extension
Functional Symbolic Trajectory	Registered succession of finite computational states

Table 23.1: The neural-network mapping is approximate. It does not define the FSM terms.

The similarity is significant because neural networks are finite physical and computational systems. Their weights, precision, memory, ordering, and state changes are available to measurement. They do not stand outside the FSM framework merely because their behaviour is described with continuous mathematics.

23.2 Limits of the mapping

The Nodal Value Function is not identical to a conventional activation function, transformer value vector, attention mechanism, or reinforcement-learning value function. It is a higher-level functional description of local endogenous valuation. A particular architecture may implement only some of its proposed components.

Latent spaces are usually encoded as finite numerical arrays and may employ Euclidean or cosine-based operations. Their functional semantic geometry may be curved, anisotropic, or discontinuous, but it should not simply be declared non-Euclidean without identifying the comparison procedure. Likewise, message passing in a graph neural network does not create order by itself. Many aggregation rules are permutation-invariant; order requires an additional registered distinction such as edge direction, position, dependency, or time.

Neural-network parameters also need to be distinguished from changing activations. Parameters may remain set during ordinary inference while contextual states change at each step. Training, inference, memory update, and corpus revision therefore occupy different temporal frames. FSM can compare them without collapsing them into one undifferentiated dynamic.

Chapter 24

Architecture and Computation

24.1 What is meant by flow through a network

Ordinary descriptions say that data or meaning flows through network layers. FSM separates the registered computational succession from the inferred entities named by that description. There is no need to posit a separately observable substance called meaning moving between nodes. What can be registered is a sequence of finite state changes:

$$\mathcal{R}_0 \prec \mathcal{R}_1 \prec \cdots \prec \mathcal{R}_n. \tag{24.1}$$

Electrical, chemical, and computational processes may be inferred and modelled from these registrations. The FSM claim is not that nothing occurs, but that the symbolic system knows occurrence through finite state mappings and their measured order.

24.2 The architecture in transformation

A frozen architecture is a registered configuration, not yet a computation. Computation lies in the ordered transformation

$$\mathcal{R}_t \xRightarrow{T_t} \mathcal{R}_{t+1}. \tag{24.2}$$

The architecture constrains the transition, and the transition may alter the functional architecture by changing activation, memory, weighting, or available context. The refined statement is therefore:

> *The registered transformation of a finite relational architecture is the computation.*

This formulation maps beyond neural networks. A written derivation, a mechanical calculator, an instrument pipeline, a biological memory system, and a conversational exchange can all be examined as finite relational architectures passing through registered states.

Part VI

A Preliminary Formal Container and Research Programme

Chapter 25

The Integrated Symbolic Cycle

25.1 From interaction to reinjection

The whole trajectory can now be assembled. An inferred exogenous field participates in a local measurement interaction:

$$\mathcal{W}^{\mathrm{exo}} \dashrightarrow M_L \longrightarrow r_t. \tag{25.1}$$

The registration is admitted as a symbol and retained:

$$r_t \xrightarrow{E_L} \sigma_t \longrightarrow \mu_t. \tag{25.2}$$

Present and memory-mapped states provide measured difference and order:

$$D_L(\widehat{\mu}_{t-1}, \mu_t) \geq \alpha_L, \qquad \widehat{\mu}_{t-1} \prec_{L,k} \mu_t. \tag{25.3}$$

The resulting trajectory is locally reconstructed and a selected symbol is prepared:

$$D_L(\sigma_i \mid \Gamma_t) \longrightarrow \widehat{\Sigma}_{\sigma_i} \xrightarrow{P_{\mathcal{R},L}} \widetilde{\sigma}_i. \tag{25.4}$$

The prepared occurrence is injected into the Dynamic Semantic Lattice, where nodal valuation changes the region:

$$I_{\mathcal{R}_t}(\widetilde{\sigma}_i) \longrightarrow \mathcal{R}_{t+1} \xrightarrow{\mathcal{V}_L} \mathcal{R}'_{t+1}. \tag{25.5}$$

A proposed continuation is measured for coherence, compressed, retained, and made available for the next cycle:

$$M_L^{\mathrm{coh}}\left(\Gamma_t, T, \mathcal{R}'_{t+1}\right) \longrightarrow c_{t+1} \xrightarrow{K_L} \sigma_{t+1} \longrightarrow \mu_{t+1}. \tag{25.6}$$

No stage is lossless by assumption. Each arrow is a compression of a realised or reconstructable finite transition carrying its own scale, uncertainty, provenance, and admissibility.

25.2 The compact cycle

The same structure can be compressed into a practical working sequence:

$$\boxed{\begin{array}{l}\text{interact} \to \text{register} \to \text{classify} \to \text{retain} \to \text{compare and order} \\ \to \text{reconstruct} \to \text{prepare and position} \to \text{inject} \to \text{revalue} \\ \to \text{transform} \to \text{measure coherence} \to \text{recompress} \to \text{reinject.}\end{array}} \tag{25.7}$$

This compression should remain recoverable into the preceding stages whenever one of its transitions is asked to carry foundational authority.

Chapter 26

Relations Among the FSM Frameworks

26.1 One framework at different functional resolutions

Finite Symbolic Mechanics provides the wider measurement-first container. Finite Symbolic Dynamics examines registered change across finite symbolic states. Finite Symbolic Logic examines admissible ordering and endogenous continuation. Finite Symbolic Algebra examines a specialised family of prepared mathematical transformations and recompressions.

The relationship is

$$
\begin{aligned}
\text{FSM} &: \text{finite registration, extent, uncertainty, provenance, and admissibility,}\\
\text{FSD} &: \text{registered state change and Functional Symbolic Trajectory,}\\
\text{FSL} &: \text{measured order, logic, and endogenous reasoning,}\\
\text{FSA} &: \text{regional algebraic preparation, transformation, and reinjection.}
\end{aligned}
\tag{26.1}
$$

These are not sealed disciplines. FSA inherits FSL; FSL is expressed through the wider language corpus; FSA may later provide tools for refining FSL notation; and every revision returns to the corpus as another symbolic trajectory. The relation is recursively coupled rather than a simple tree.

26.2 The role of the Dynamic Semantic Lattice

The Dynamic Semantic Lattice provides a common finite relational model across the frameworks. Within FSM it situates registered occurrences. Within FSD it supports changing state configurations. Within FSL it supplies the region in which ordering and reasoning occur. Within FSA it supplies the prepared mathematical field in which substitutions and transformations are measured.

The lattice is therefore not an additional metaphysical layer. It is a modelling container through which the finite relational conditions already present in registration, memory, language, logic, and algebra can be placed into one coherent view.

Chapter 27

Open Formal Questions

27.1 The nodal value and its output

The Nodal Value Function requires a usable formal representation. A universal scalar is unlikely to preserve relevance, uncertainty, provenance, scale, and admissibility simultaneously. The first development should treat nodal value as a structured local state or vector whose components can vary by region.

The distinction between an occurrence-node and a compressed type-node also requires explicit operators. A compression rule must state which occurrence differences it suppresses, how long the equivalence remains admissible, and what event forces the type-node to be unfolded again.

27.2 Memory, time, and provenance

Memory mappings require separate temporal coordinates. Registration time, symbolic trajectory time, computation time, inferred endogenous time, inferred exogenous time, and dispositional publication order may differ. A sequence can be exact in a machine log while unresolved in the interaction the log purports to describe.

The relationship $\widehat{\mu}_{t-1} \sim_H \mu_{t-1}$ therefore requires a calculus of provenance strength, compression, and uncertainty. A memory relation should be able to state what has been retained, what has been transformed, and what is unavailable.

27.3 Regional boundaries and bridges

The boundary of a functional symbolic regional container remains partly diagnostic. Future work must identify operational tests for when a symbol has crossed into another scale, domain, or reconstruction basin. A bridge should declare which relations it preserves rather than simply reuse the same notation in the new region.

This is particularly important for the relation between formal algebra and measurement. A transformation can be impeccable within a formal region while its terms refer ambiguously to counted, measured, or set numbers. The bridge back to registration must carry those formation differences.

27.4 Algebraic law and trajectory preservation

Associativity, commutativity, distributivity, identity, inversion, and cancellation should be reconstructed one at a time. The question is not merely whether each law is true, but which trajectory differences it suppresses and under what local conditions those differences remain irrelevant.

This programme may reveal a hierarchy of laws. Some may preserve terminal inscriptions, others measured values, others admissible substitutions, and still others provenance-bearing trajectories. Conventional algebra often uses one equality sign across these levels. FSA should make their relation inspectable without making ordinary calculation unusably cumbersome.

Chapter 28

Empirical and Computational Directions

28.1 Proof and finite implementation

Proof assistants provide a natural environment in which abstract derivation can be compared with parsing, memory, execution, and verification trajectories. Error injection near token, storage, timing, and precision boundaries could show where implementation differences become logically consequential.

A proof line can also be stored at multiple resolutions: as visible text, syntax tree, machine state, execution trace, and compressed theorem name. Comparing these resolutions would provide a practical study of occurrence-node and type-node relations.

28.2 Language models and reconstruction basins

Language models allow the same compressed prompt to be prepared through different provenance paths. One can measure whether additional context changes nodal valuation, basin selection, and terminal continuation. Candidate measures include branch density, return, context sensitivity, provenance divergence, uncertainty growth, and the cost of restoring compressed context.

The TBT programme offers a particularly relevant implementation because it treats language as trajectory-sensitive and foregrounds delay, memory, and reconstruction. The Dynamic Semantic Lattice and Nodal Value Function may provide a clearer conceptual vocabulary for describing how a prepared token occurrence enters a local state, changes relational weighting, and redirects continuation.

28.3 Human reconstruction and regional compression

Human studies could compare how readers reconstruct the same mathematical expression after different explanatory run-ins. A symbol prepared through measurement, formal definition, visual analogy, or narrative history may enter a different local basin even when its terminal inscription is the same.

Such studies would not seek one universal hidden semantic geometry. They would examine measurable differences in continuation, recall, substitution, error, and transfer across local containers. The reader becomes an active endogenous constructor rather than a passive receiver of completed meaning.

Chapter 29

Objections and Clarifications

29.1 "This merely adds context to algebra"

The framework does more than append contextual notes. It changes the foundational priority. The algebraic term is not treated as a complete object to which context is later added. It is treated as a compression whose function depends upon a reconstructable trajectory. Context is constitutive of operability, even where repeated practice makes it invisible.

29.2 "The framework makes calculation impossible"

FSA does not require total decompression before every operation. Such a demand would contradict finite communication. It requires sufficient recoverability when a suppressed distinction becomes consequential. Ordinary algebra can remain compact inside a stable regional container.

29.3 "A formal equality needs no physical history"

Inside a declared formal calculus, many occurrence differences are intentionally irrelevant. FSM accepts the usefulness of that compression. It asks how the calculus is realised, read, verified, and transferred to measurement, and what happens when a previously irrelevant difference crosses the operative boundary.

29.4 "The lattice reifies semantic space"

The Dynamic Semantic Lattice is a model of finite relational registrations. It does not assert an independently existing smooth semantic space. Nodes, weights, position, curvature, and basin are operational descriptions whose admissibility depends upon a stated procedure and locality.

29.5 “Endogenous coherence is circular”

Reasoning about reasoning is recursive, but recursion need not imply a vicious circle. A later finite trajectory can measure an earlier one under another local frame. The later judgement is neither final nor external; it carries its own provenance and can itself become the subject of another measurement.

Chapter 30

Conclusion: Algebra Within the Flow

The work began by asking what an FSM algebra might be. The initial temptation was to retain familiar algebraic objects and add extent, uncertainty, provenance, scale, and admissibility. The developing trajectory showed that this did not go far enough. A symbol does not first exist as a complete object and later acquire conditions. It is a finite occurrence whose function depends upon the compressed symbolic paths through which it is recognised, classified, reconstructed, positioned, and allowed to operate.

The argument then returned to the measurement boundary. An exogenous continuation is inferred, while the symbolic system receives finite registrations. These registrations are admitted into symbolic classes and retained through memory mappings. Flow becomes available when present and memory-mapped states are measurably distinguished and ordered. The discrete states belong to the endogenous map; they do not by themselves establish the final constitution of the inferred exogenous field.

The available states and their changing relations form a Dynamic Semantic Lattice. Its nodes are unique finite occurrences rather than identical ideal points. Their values are locally measured through Nodal Value Functions that take account of neighbourhood, weighting, memory, provenance, uncertainty, scale, and direction. A prepared symbol is reinjected into the lattice, changes the regional relations, and participates in another Functional Symbolic Trajectory.

The lattice is held within the wider Alphon and corpus of language. Mathematics is a strongly stabilised regional basin inside that corpus. Its compact notation succeeds because a reader or machine reconstructs a large undeclared container of definitions, equivalences, ordering conventions, and permitted transformations. The words-within-words condition prevents total contextual display, but it does not prevent sufficient local recovery.

Finite Symbolic Logic supplies the conditions under which the flow becomes admissibly ordered and reasoned. Finite Symbolic Algebra inherits those conditions and concentrates upon a specific endogenous cycle: selection, reconstruction, preparation, positioning, transformation, coherence measurement, recompression, and reinjection. Equality becomes permission to substitute one compressed trajectory for another under declared local conditions. Algebraic laws become regional coherences whose stability depends upon which trajectory differences remain below the operative distinction boundary.

The resulting framework can be held in one central proposition:

> *An inferred exogenous interaction is mapped through measurement into finite registered states. Memory makes prior mappings presently available for comparison. Registered difference and order produce finite symbolic flow. The changing relational organisation of those occurrences forms a Dynamic Semantic Lattice. Symbols drawn from the wider Alphon are reconstructed, prepared, and finitely positioned within regional containers. Nodal Value Functions measure their local contribution and admissible continuations. Their transformation and reinjection produce a Functional Symbolic Trajectory whose coherence is measured endogenously. Finite Symbolic Algebra studies this process within mathematical regions.*

This is not yet a complete algebra. It is the point from which an algebra can now proceed without losing the wider FSM trajectory that made the question possible. The next work can become more local and technical because the location of that work has been retained.

Appendix A

Provisional Notation

Table A.1: Working notation used in the monograph.

Symbol	**Working interpretation**
$\mathcal{W}^{\mathrm{exo}}$	Inferred exogenous interactional field.
M_L	Local measurement interaction.
r_t	Finite registration.
E_L	Local encoding or classification operation.
σ_t	Finite symbolic occurrence.
μ_t	Memory-mapped state.
$\widehat{\mu}_t$	Presently reconstructed or partially retained memory relation.
α_L	Operative Alphonic limit in locality L.
Γ_t	Finite memory-bearing trajectory.
$\mathcal{A}$	Available Alphon or symbolic formation field.
$\mathcal{C}$	Available corpus of registered symbolic trajectories.
$\mathcal{R}_t$	Local Dynamic Semantic Lattice state or regional container.
N_t	Available nodal occurrences.
W_t	Finite relational weighting structure.
$\mathcal{M}_t$	Available finite memory field.
$P_{\mathcal{R},L}$	Preparation and regional positioning operation.
$\widetilde{\sigma}_i$	Symbol occurrence prepared for regional use.
$\mathcal{V}_{i,L}^{(t)}$	Nodal Value Function at node i, locality L, and state t.
$\mathbf{v}_i^{(t)}$	Structured local nodal valuation.
$\mathcal{D}_i^{(t)}$	Locally admissible continuation region.
M_L^{coh}	Endogenous coherence measurement.
$\mathbf{C}_L(T)$	Vectorial local description of transformation coherence.
K_L	Local trajectory compression or recompression operation.
$\sim_A$	Admissible equivalence under symbolic class A.
$\sim_{\mathcal{O},L}$	Operational substitutability under operation $\mathcal{O}$ in locality L.
$\prec_{L,k}$	Registered precedence in locality L under coordinate k.

Appendix B

Glossary

Alphon
The epochal and operationally available field of distinguishable symbolic formation resources, including words, numerals, operators, diagrams, codes, and other admitted marks.

Alphonic Limit
The operative minimum first-order distinction that can be registered and carried as a symbol under stated conditions.

Compression
The construction of a shorter symbolic representation by suppressing or summarising parts of a longer trajectory. Compression is not identity with the source trajectory.

Dynamic Semantic Lattice
A finite, irregular, memory-bearing, and changing relational organisation of unique symbolic occurrences.

Endogenous Measured Coherence
A finite local measurement of whether a proposed transformation and continuation preserves sufficient distinction, order, provenance, scale, uncertainty, and admissibility to continue within a region. The result re-enters the field it measures.

Finite Symbolic Algebra
The endogenous preparation, positioning, transformation, coherence measurement, recompression, and reinjection of finite symbolic trajectories within a locally admissible mathematical region.

Finite Symbolic Dynamics
The description of registered changes, extensions, branches, returns, and transformations across finite symbolic states.

Finite Symbolic Logic
The measurement-first framework in which logic is finite admissible symbolic ordering and reasoning is the endogenous measurement and continuation of that flow.

Functional Symbolic Regional Container
A finite locally stabilised context that prepares symbols, supplies admissible relations, and constrains possible continuations without displaying the whole wider corpus.

Functional Symbolic Trajectory
A finite ordered and provenance-bearing passage through changing symbolic states and transformations.

Memory-Mapped State
A finite present state that retains or reconstructs a provenance relation to an earlier registration sufficiently for comparison and ordering.

Nodal Value Function
The local operation through which the present relational value and admissible continuations of a node are measured and updated within a Dynamic Semantic Lattice.

Real Finite Positioning
The placement of a symbol through measurable finite relations within a regional container, rather than assignment to an assumed unextended semantic point.

Reinjection
The return of a reconstructed, prepared, transformed, or recompressed symbol into the active regional field, where it changes the conditions of subsequent continuation.

Symbolic Flow
The measurably ordered reconstruction of present and memory-mapped discrete states. It does not require an independently transported substance called meaning.

Type-Node
A practical compression across finite occurrence-nodes admitted under a common symbolic class. Its operational stability does not make it identical to the occurrences from which it was formed.

Appendix C

A Compact Continuation Marker

The following paragraph is intended as a trajectory marker from which later work can resume without reconstructing the entire monograph.

> *Finite Symbolic Algebra begins before the visible operation. An inferred exogenous interaction is mapped through measurement into finite registrations, which are classified as symbols and retained through memory-mapped states. Measured distinction and order make symbolic flow available. The currently active occurrences, relations, and memories form a Dynamic Semantic Lattice within the wider Alphon and corpus of language. A symbol is a compression of reconstructable trajectories and must be prepared and finitely positioned before reinjection into a regional container. Its Nodal Value Function measures what it can presently do from that position. Transformation alters the region, and endogenous coherence measurement determines whether the resulting compression is admissible for continuation. Equality records local substitutability between different trajectories rather than literal identity. The result is retained and reinjected, making algebra a finite symbolic dynamical cycle within Finite Symbolic Logic and the wider FSM framework.*

Author’s Closing Note

This monograph records a point of convergence rather than an endpoint. Its compressed formulations emerged through a longer trajectory involving measurement, finite symbols, memory, language, logic, dynamics, neural computation, algebra, and repeated correction at the boundary between inference and registration. The apparent compactness of the final schema should not conceal that supporting history.

The reader will necessarily reconstruct another trajectory from the text, drawing upon a different memory field and wider corpus. That difference is not external to the subject. It is one of the finite symbolic processes the work has attempted to preserve. The value of the monograph will lie not in freezing the terminology, but in giving later inquiry a sufficiently coherent region from which to continue.