On Finite Mathematical Objects

Finite Operational Unfolding and Spherical Realisation

in Finite Symbolic Mechanics

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August 2026

Prefatory Note

This monograph records a conjecture, the trajectory by which it was approached, and the correction that returned it to the foundational commitments of Finite Symbolic Mechanics. The inquiry began with the apparently simple proposal that any finite symbolic trajectory, including a mathematical proof or an unfolded higher-order construction, might be mapped onto a finite sphere. The proposal did not arise as an isolated geometrical image. It emerged from a longer investigation into finite inscription, the Alphonic Limit, uncertainty, provenance, admissibility, mathematical compression, and the physical extent of every symbol by which mathematics is performed.

An early bridge into classical mathematics appeared through elliptic functions, complex tori, Riemann surfaces, sheets, branch points, and maps onto the Riemann sphere. The bridge was illuminating but was initially crossed too quickly. The completed classical surface was treated as though it were already the geometry required by FSM. In particular, the classical language of sheets and branch points began to occupy the proposed finite sphere before those objects had themselves been operationally unfolded. This repeated the very compression that the inquiry was intended to expose.

The decisive correction was to re-establish the FSM constraint. A classical mathematical object cannot be transferred directly into the proposed FSM geometry. It must first be unfolded into the finite, sequential symbolic operations through which it is constructed, calculated, invoked, or checked. This unfolding produces a time-ordered series of finite symbolic states. Each state has positive extent, uncertainty, provenance, and conditions of admissibility. The resulting construction is described in this monograph as a *nodal rope*: a flexible trajectory of finite beads joined by finite operational links.

The sphere enters only after this unfolding. The question is not whether a completed elliptic surface can be projected onto another completed surface. The question is whether the finite operational rope can be folded onto a finite spherical construction in three-dimensional space while preserving every distinction required for its reconstruction. A failure to achieve that construction is not to be concealed. It is an admissible result capable of exposing either an incomplete unfolding, an inadequate mapping rule, or a classical operation for which no finite spherical realisation has yet been found.

Abstract

Classical mathematics commonly presents matrices, functions, curves, surfaces, proofs, and higher-dimensional constructions as completed objects. Their notations are nevertheless finite symbolic compressions of procedures that must be enacted through ordered registrations and transformations. Finite Symbolic Mechanics begins by reopening these compressions.

This monograph introduces the *bead* as a finite symbolic state possessing positive three-dimensional extent, uncertainty, provenance, and admissibility conditions. A finite calculation or proof is represented as a *nodal rope*: a time-ordered sequence of beads joined by finite operational links. Matrices provide the initial prototype because their apparently higher-dimensional operations can be unfolded into nested finite sequences. Elliptic functions, sheets, branch points, and Riemann surfaces provide a stronger test by revealing how completed classical geometries compress branch selection, continuation, equivalence, and closure instructions.

The central conjecture states that every admissible finite symbolic trajectory may possess a finite spherical realisation in three-dimensional space from which its ordered operations, distinctions, uncertainty, and required provenance can be reconstructed. An unconstrained placement of a finite chain is distinguished from a strong reconstructive realisation. The latter is proposed as one possible form of FSM proof: a finite and inspectable construction demonstrating that the symbolic operations of a classical proof possess an admissible three-dimensional realisation. The conjecture remains open. No intrinsic proof-genus, ontological Riemann surface, or correspondence with physical gravitational mass is asserted.

Statement of Epistemic Status

The finite extent of inscriptions, the sequential enactment of calculations, and the need to preserve uncertainty and provenance are foundational commitments of FSM. Operational unfolding is the proposed method by which compressed classical objects are decompressed into finite symbolic trajectories. The spherical mapping of the resulting nodal rope is the principal conjecture. The proposal that a successful, recoverable three-dimensional construction may constitute an FSM realisation proof is an experimental methodological proposal rather than an established theorem.

The monograph therefore separates what is declared, what is derived, what is conjectured, and what remains to be measured. Classical objects are described accurately within their own symbolic frameworks, but they are not granted independent measurable existence merely because their formal definitions are coherent. Conversely, the inability to produce an FSM realisation does not refute a classical theorem within its own rules. It records a difference between classical formal admissibility and finite symbolic realisability.

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Part I

The Finite Condition Before the Mathematical Object

Chapter 1

The Completed Object and the Omitted Trajectory

1.1 The growth of mathematical objects

The historical development of mathematics may be read as a progressive stabilisation of symbolic entities. Number, point, line, function, matrix, field, manifold, and proof became increasingly effective nouns. Each could be placed within a sentence, subjected to a rule, and carried from one calculation to another. This stabilisation enabled extraordinary compression. A long sequence of operations could be gathered into a single sign, and the sign could then participate in a still larger construction.

Compression was not an error introduced into mathematics. It was a condition of mathematical development. No practitioner could restate the full history of number, equality, multiplication, or proof whenever a calculation was undertaken. The capacity to suppress established trajectories enabled practical repetition, teaching, and collective extension. Yet the success of the compression also concealed the physical and historical work required to sustain it.

A matrix may be named by a capital letter. A function may be treated as a completed mapping. An elliptic curve may be given by a short equation. A theorem may be invoked by name. In each case the compact expression is supported by a wider lattice of definitions, permitted transformations, previous proofs, inscriptions, instruments, and conventions. The visible object is the terminal compression of that support.

1.2 Nouns, rules, and the appearance of completion

The noun form encourages the reader to encounter the mathematical entity as already present. The phrase ‘the complex plane’ appears to name a completed field of positions. The phrase ‘the Riemann surface’ appears to name a surface available for inspection. The phrase ‘the proof’ appears to name a stable object that can be possessed, cited, and transmitted. The underlying verbs—writing, selecting, calculating, comparing, revising, accepting, and reconstructing—move into the background.

Finite Symbolic Mechanics does not prohibit the noun. It asks that the noun retain a route back to the operations that sustain it. A supported mathematical noun may be represented schematically as

$$\text{mathematical object} = \text{Compress}\left(\text{definitions} + \text{operations} + \text{provenance} + \text{admissibility}\right).$$

The equality here is not an identity between an ideal object and a finite list. It is a reminder that the usable symbolic entity depends upon an omitted trajectory.

1.3 The contemporary density of exposition

The present epoch multiplies mathematical and scientific exposition at great speed. New models, conjectures, diagrams, and theoretical vocabularies appear continuously. Their volume can create an expectation field in which repeated terminology acquires apparent weight before its measurement trajectory has been inspected. The number of documents is not a measurement of the depth or independence of their foundational commitments.

This problem was visible in the analysed gravity article in which the word 'pattern' moved from a description of propagating organisation to a causal entity capable of carrying momentum, flowing inward, and dropping an apple. The semantic path was coherent because examples and neighbouring words made the central term increasingly dense. Yet the density was partly compressive. Distinct mechanisms were gathered beneath a shared word without a complete declaration of their differences.

The same risk applies to the present monograph. Words such as 'sphere', 'sheet', 'branch', 'closure', and 'proof' carry established classical trajectories. They cannot be transferred into FSM merely because their associations are suggestive. Each must be reopened.

Chapter 2

Finite Symbolic Mechanics Begins Before the Object

2.1 The Generonic Boundary

FSM begins before formal mathematical objects. It asks how a finite distinction becomes available to a symbolic system. The Generonic Boundary names the operational transition at which an interaction becomes a finite registration capable of storage, comparison, transmission, and further transformation. The registration is not the interaction in its entirety. It is the finite symbolic result generated under particular conditions.

The foundational sequence is written as

$$\begin{aligned}\text{interaction} \longrightarrow \text{Generonic registration} \longrightarrow \text{finite distinction}\\ \longrightarrow \text{symbol} \longrightarrow \text{relation} \longrightarrow \text{trajectory.}\end{aligned}$$

Every later mathematical construction depends upon this passage, even when its formal language begins after the symbols have already been admitted.

2.2 The finite symbol

Definition 2.1 (Finite symbol)**.** *A finite symbol is an interactionally produced, positively extended registration that can participate in a finite symbolic trajectory. It carries a distinguishable form, a production history, an uncertainty region, and conditions governing its admissible use.*

The finite symbol is not an extensionless point. A mark on paper, a stored machine value, a state in an electronic register, and a displayed character all require finite interactional volume under uncertainty. Their apparent two-dimensionality is a projection of a three-dimensional production and storage process.

Finite measured numerical values are treated as primary finite symbolic objects. They are not described as inferior approximations to completed real numbers waiting beyond measurement. A classical real may function as an endogenous rule-governed construction. A measurable number is the finite result actually registered under declared conditions.

2.3 The Alphon, the Nexil, and the Alphonic Limit

The *Alphon* is the finite admissible symbolic set available to a particular system. A *Nexil* is an individual finite member or operative symbolic unit within that set. The Alphonic Limit is the smallest currently admissible level at which a distinction can be registered and sustained. It is dependent upon instrument, condition, epoch, and consensus, and it remains revisable.

At this limit, the minimum registration is provisionally represented by an isotropic uncertainty-containment sphere. This does not assert that an electron, word, number, or physical entity is literally a tiny ball. The sphere represents the minimum non-directionally privileged containment of a finite distinction under uncertainty.

2.4 Uncertainty and the Spherical Uncertainty Distribution

The Alphonic sphere and the Spherical Uncertainty Distribution are related but distinct. The Alphonic sphere concerns a minimum local containment. The SUD concerns the admissible spread, interaction, and packing of uncertainty among multiple finite registrations.

If a finite state is denoted by B_i, its local uncertainty may be represented as

$$U_i = B(c_i, r_i), \qquad r_i \geq \alpha,$$

where c_i is a finite registered centre, r_i is an uncertainty radius, and α is the declared Alphonic resolution. Actual instruments may generate anisotropic uncertainty. The isotropic sphere is therefore a provisional reference geometry rather than an imposed description of every realised registration.

2.5 Provenance and admissibility

Two visibly identical symbols need not have equal trajectories. They may arise from different instruments, calculations, assumptions, or histories. Provenance is therefore part of the finite state rather than optional commentary added after the result.

An FSM transition may be written

$$P \longrightarrow Q \mid (C, \alpha, H, \delta),$$

where C records admissibility conditions, α records the operative Alphonic limit, H records provenance, and δ records uncertainty. The expression does not replace the full transition history. It marks the conditions that classical implication commonly suppresses.

Chapter 3

Compression and Operational Unfolding

3.1 The compressed expression

Let $\mathcal{O}$ denote a classical mathematical object and let $E(\mathcal{O})$ denote its finite written expression. The expression is not treated as the complete object. It is treated as an instruction-bearing compression:

$$E(\mathcal{O}) = \mathrm{Compress}\left(\mathcal{U}_\alpha(\mathcal{O})\right),$$

where $\mathcal{U}_\alpha(\mathcal{O})$ is the finite operational unfolding performed at resolution α.

The unfolding does not reproduce an ideal continuum. It reconstructs the finite operations used in a particular construction, calculation, or proof. The distinction between the rule, an execution of the rule, and the ideal totality implied by the rule must remain visible:

$$\text{rule} \neq \text{finite execution} \neq \text{completed implied totality.}$$

FSM can register a finite rule and a finite execution. It does not thereby instantiate every member of the indefinitely extensible domain to which the rule may refer.

3.2 Finite Procedural Unfolding

Definition 3.1 (Finite Procedural Unfolding)**.** *Finite Procedural Unfolding is the declared decomposition of a compressed symbolic expression into the finite registrations, intermediate states, transformations, comparisons, and termination conditions required for a particular enactment of that expression.*

Its general trajectory is

$$\begin{aligned}\text{compressed notation} &\longrightarrow \text{parameter registration} \longrightarrow \text{stored state}\\ &\longrightarrow \text{ordered operation} \longrightarrow \text{intermediate registration}\\ &\longrightarrow \text{termination test} \longrightarrow \text{reported result.}\end{aligned}$$

Each arrow must eventually be represented by a finite operation rather than an ideal line. Each state must eventually be represented by a finite registration rather than an extensionless point.

3.3 Unfolding and recursion

A proof frequently invokes an earlier lemma as a single node. The lemma may itself invoke definitions and prior results. Operational unfolding can therefore proceed recursively, but no finite document can contain every supporting trajectory without limit. The unfolding must declare its finite horizon.

An invoked lemma may be retained as a compressed bead provided that its provenance, admissibility status, and route to further expansion are preserved. The aim is not to eliminate compression. It is to prevent compression from appearing costless or self-grounding.

3.4 The editorial rule

Every unfamiliar classical object in this monograph is approached through four layers:

$$\boxed{\text{accessible picture} \longrightarrow \text{classical role} \longrightarrow \text{finite calculation} \longrightarrow \text{FSM admissibility}}.$$

This is not merely a pedagogical convenience. It prevents the classical completed object from entering the finite framework before its operational content has been exposed.

Part II

Reopening Classical Mathematical Objects

Chapter 4

The Matrix as Prototype

4.1 The completed array

Classically, a matrix is presented as an indexed array. If

$$C = AB,$$

the notation appears to relate three completed objects. The entry rule

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

reveals some of the concealed process but continues to compress the acts of retrieval, multiplication, accumulation, comparison, and storage.

4.2 A finite operational reading

For fixed finite indices i and j, the calculation begins by registering an initial accumulation state. It retrieves a_{i1} and b_{1j}, performs a finite multiplication, registers the result, updates the accumulation, and tests whether another k-state remains. The procedure continues until the declared finite index is exhausted. The value c_{ij} is then registered, and the procedure advances to the next admissible index pair.

The compressed summation becomes

$$s_0 \longrightarrow s_1 = s_0 + a_{i1} b_{1j} \longrightarrow s_2 = s_1 + a_{i2} b_{2j} \longrightarrow \cdots \longrightarrow c_{ij} = s_n.$$

Each multiplication and addition is itself a finite symbolic trajectory. The equation displays their stabilised order but does not stand outside their production.

4.3 Higher order without higher physical dimension

A matrix may be displayed as a two-dimensional table, but its computational role is carried by indexing rules. A tensor adds further indices. The additional order is implemented through nested selection, return, and storage instructions. It does not require a corresponding number of physical spatial dimensions.

This distinction is written

$$\text{declared mathematical dimension} \neq \text{physical dimension of inscription.}$$

A finite use of a ten-index object may require extensive operational depth, but every value and instruction remains physically instantiated through finite three-dimensional machinery.

4.4 The matrix as an initial rope

The complete execution trace of the multiplication can be written

$$B_0 \xrightarrow{E_0} B_1 \xrightarrow{E_1} \cdots \xrightarrow{E_{m-1}} B_m.$$

The states B_i are provisional beads, while the operations E_i are provisional links. The matrix is therefore the first clear example of a classical completed object being reopened as a nodal rope.

Chapter 5

Higher-Order Objects as Nested Trajectories

5.1 Operators, tensors, and eigenvectors

Classical notation can gather a large family of operations into a named operator. A tensor can gather a nested system of indexed relations. An eigenvector can be described as an object satisfying a compact equation. In each case the symbolic compression is useful, but the finite use of the object requires parameter registration, storage, transformation, comparison, and a declared criterion of acceptance.

For example, the classical relation

$$Av = \lambda v$$

does not by itself display the finite path by which A, v, and λ are represented or tested. An FSM unfolding must show how the components are retrieved, how the matrix action is enacted, how the comparison is made, and under what finite uncertainty the two sides are admitted as equivalent.

5.2 Operational depth

The phrase 'higher dimension' often compresses operational depth into spatial language. Within FSM, the initial question is not how many ideal axes an object inhabits. It is how many finite distinctions, nested references, and ordered transformations are required for its use.

This motivates the provisional separation

$$\text{spatial extent} \quad \text{from} \quad \text{operational order} \quad \text{from} \quad \text{symbolic depth.}$$

These quantities may interact, but they are not synonyms.

5.3 The finite claim

The claim made here is limited. A finite symbolic use of a higher-order object can be decomposed into finite registrations and operations enacted in three-dimensional physical space. This does not prove that every classical structure has been ontologically reduced to three dimensions. It identifies the finite trajectory through which that structure enters a calculation.

Chapter 6

The Complex Plane as Coordinate Permission

6.1 The familiar picture

A complex number is written

$$z = x + iy.$$

The familiar picture places x and y upon perpendicular axes. The complex plane is then treated as the completed field of all such ordered pairs.

6.2 The classical role

Within classical mathematics the complex plane supports algebraic operations, analytic continuation, functions, contours, and transformations. Its completed form allows a practitioner to reason globally about relations that no finite calculation enumerates one by one.

6.3 The finite execution

In a finite calculation a particular complex state consists of finite representations of x and y, together with stored rules governing i, addition, multiplication, and comparison. The calculation visits a finite set of states:

$$(x_0, y_0) \longrightarrow (x_1, y_1) \longrightarrow \cdots \longrightarrow (x_m, y_m).$$

The complex plane is the compressed relational permission under which these paired states are interpreted. It is not necessary to instantiate the complete continuum in order to enact the finite trajectory.

6.4 Finite-symbolic admissibility

The complex notation is admissible as a finite rule-governed symbol system. Its use in an FSM proof requires the finite states and operations actually invoked to be unfolded. The ideal plane cannot be silently identified with a physical sheet or with the proposed finite sphere.

Chapter 7

Period Lattices, Elliptic Functions, and Elliptic Curves

7.1 The period lattice

Classically, two periods generate a lattice

$$\Lambda = \mathbb{Z}\omega_1 + \mathbb{Z}\omega_2.$$

The notation declares an indefinitely extensible set of translations. A finite calculation does not construct every member of this set. It registers finite generators, integer-index rules, and the finite translations actually visited.

The operational content of the lattice is therefore closer to

$$(\omega_1, \omega_2) + (m, n) \longrightarrow m\omega_1 + n\omega_2,$$

where m and n are finite symbolic values used in a particular calculation. The completed lattice remains a classical rule-governed totality.

7.2 Elliptic functions

An elliptic function is classically doubly periodic:

$$f(z + \lambda) = f(z), \qquad \lambda \in \Lambda.$$

In a finite execution, periodicity becomes a rule for comparing finitely represented states. The system calculates or registers z, calculates a selected λ, forms $z+\lambda$, evaluates the finite expressions required for $f(z)$ and $f(z+\lambda)$, and applies a declared equivalence condition.

The equation compresses that trajectory. It does not make every value of the function simultaneously present.

7.3 Elliptic curves

A classical elliptic curve may be written

$$y^2 = 4x^3 - g_2x - g_3.$$

The completed-object reading treats the curve as the locus of all points satisfying the equation. The finite operational reading begins by registering g_2, g_3, and a finite x-state, calculating the polynomial expression, generating or selecting a finite y-state, recording any branch condition, and applying a finite termination rule.

The curve constrains the rope. It is not itself the rope. Its global geometric interpretation may guide classical reasoning, but the FSM object is the finite sequence through which the constraint is used.

Chapter 8

Sheets, Branch Points, and Infinity

8.1 The visual sheet

For an expression

$$y^2 = P(x),$$

an ordinary finite value of x may be associated classically with two values:

$$y = +\sqrt{P(x)} \qquad \text{and} \qquad y = -\sqrt{P(x)}.$$

The classical visualisation describes these alternatives as two sheets lying above the x-plane. A locally continuous movement follows one choice until the geometry requires a change in the description.

8.2 The sheet as an operational state

No finite calculation evaluates an entire sheet. It registers a finite x_i, calculates a finite $P(x_i)$, selects or continues an admissible y_i, records the current branch state, and advances:

$$x_i \longrightarrow P(x_i) \longrightarrow y_i \longrightarrow b_i \longrightarrow x_{i+1}.$$

The sheet is therefore unfolded as a branch-state and continuation rule carried by the operational trajectory. It is not transferred into FSM as a material layer.

8.3 The branch point

A branch point is classically a location at which branches meet or cease to remain locally distinguishable under the chosen representation. In the finite rope it becomes an event at which the branch record, distinction condition, or continuation rule changes:

$$\text{distinct continuations} \longrightarrow \text{locally merged state} \longrightarrow \text{new continuation rule}.$$

Whether such an event later becomes an alignment node on a spherical realisation is an experimental question. Its location cannot be imported directly from the classical drawing.

8.4 The point at infinity

Classical compactification may adjoin a point at infinity. FSM cannot treat this as a measurable bead at an infinitely distant location. It must unfold the finite rule through which the symbol functions. This may involve a reciprocal transformation, a projective coordinate rule, a limiting instruction, or another finite symbolic construction.

The finite registration is therefore not infinity itself. It is the finite operation by which an accepted mathematical system marks and uses a closure condition.

Chapter 9

The Critical Correction

9.1 The premature bridge

The classical theory of elliptic curves supplies relations among period lattices, complex tori, sheets, branch points, and the Riemann sphere. These relations appeared to give the spherical conjecture immediate classical support. The torus seemed to provide closure, while the branched projection seemed to concentrate complexity into a small number of special points.

This interpretation contained a category error. The classical surface was treated as though it were already the finite spherical geometry of FSM. The sheets were treated as though they were finite shells, and branch points were treated as though they were pre-existing spherical nodes.

9.2 Restoring the finite constraint

The correction restores the required sequence:

$$\boxed{\begin{array}{c}\text{classical compressed object} \longrightarrow \text{finite operational unfolding} \\ \longrightarrow \text{nodal rope} \longrightarrow \text{three-dimensional spherical test}\end{array}}.$$

The Riemann surface may constrain the classical rules being unfolded. It is not the object mapped directly to the FSM sphere.

9.3 What remains useful

The classical material remains valuable because it identifies operations that the rope must carry: periodic comparison, branch choice, continuation, equivalence, closure, and the handling of exceptional states. The correction therefore does not abandon the classical bridge. It relocates it. Classical mathematics supplies compressed operational constraints; FSM asks how those constraints are finitely enacted.

9.4 What is not claimed

No intrinsic genus is assigned to a proof. The Riemann–Hurwitz relation is not treated as a measure of proof complexity. Symbolic curvature is not identified with gravitational curvature, and no relation between proof structure and gravitational mass is proposed. These ideas remain examples of how quickly a shared word can transfer permission between distinct semantic domains.

Part III

The Bead, the Rope, and the Sphere

Chapter 10

The Bead as a Finite Symbolic State

10.1 Why the bead is not merely a token

A bead is the smallest state admitted to the present construction, but it is not a bare character taken in isolation. A symbol can function only because it is registered at a finite stage, under a finite history, with a finite degree of uncertainty, and within an available field of consensual distinctions. The bead therefore records a symbol together with the conditions under which that symbol can participate in a trajectory.

For a finite resolution floor $\alpha > 0$, write the i-th bead as

$$B_i = (\sigma_i, t_i, H_i, U_i, C_i, \Gamma_i). \tag{10.1}$$

Here σ_i is the registered symbol or finite symbolic state, t_i is its place in the operational order, H_i is the accessible history required at that place, U_i is an uncertainty neighbourhood, C_i is the relevant consensual permission, and Γ_i is the local set of admissible continuations. None of these entries is presumed exact beyond the alphonic limit.

10.2 The alphonic neighbourhood

The pictorial bead is a finite sphere rather than a mathematical point. A convenient local notation is

$$U_i = \mathcal{B}(c_i, r_i), \qquad r_i \geq \alpha, \tag{10.2}$$

where c_i is a registered centre and r_i is no smaller than the current resolution floor. This notation does not claim that cognition contains literal Euclidean balls. It supplies a finite geometric carrier for the assertion that every registration occupies a non-zero neighbourhood and that distinctions below α are unavailable to the present measurement.

The centre is therefore not an exact semantic essence. It is a representative location under the chosen construction. Two registrations may be treated as the same when their differences fall within the accepted uncertainty and consensus conditions, while apparently identical inscriptions may remain distinct when their histories or continuation permissions differ.

10.3 Lexils and Nexils

The symbolic component σ_i may be a Lexil, such as a word or a developed linguistic unit, or a Nexil, such as an operational sign whose function is primarily relational. The distinction is not absolute. A multiplication sign can acquire explanatory language, and a word can act as a tightly constrained operator. What matters is that the current role is declared rather than silently inferred.

In the rope, Lexils commonly carry comparatively broad semantic neighbourhoods, while Nexils commonly narrow the admissible next operations. This is a tendency rather than a definition. A formal term may be more tightly constrained than an overloaded equality sign, and a familiar word may function almost deterministically inside a stipulated calculus.

10.4 Local semantic uncertainty

Semantic uncertainty is not represented by a single scalar unless a particular experiment justifies that reduction. At minimum it may include uncertainty about reference, operational role, inherited context, branch state, and permissible continuation. One may therefore write

$$U_i = U_i^{\text{ref}} \times U_i^{\text{role}} \times U_i^{\text{hist}} \times U_i^{\text{branch}} \times U_i^{\text{next}}, \tag{10.3}$$

with the warning that the product sign is an organisational device until the factors and their interactions are experimentally specified. The useful claim is modest: uncertainty belongs to the bead and cannot be recovered reliably from the visible symbol alone.

Chapter 11

The Link as a Finite Operation

11.1 From adjacency to enacted relation

Two beads lying next to one another do not yet form a calculation. A link records the finite operation by which one registered state gives permission for another. Let

$$E_i = \left(o_i, \Delta t_i, H_i^E, U_i^E, C_i^E\right) \tag{11.1}$$

denote the link from B_i to B_{i+1}. The term o_i identifies the enacted operation, Δt_i its finite ordering interval, H_i^E the history required to apply it, U_i^E its uncertainty, and C_i^E its consensual warrant.

This link may be a formal inference, a matrix multiplication and addition, a substitution, a choice of branch, a retrieval of a definition, or an ordinary linguistic transition. The rope does not assume that all links are of one kind. It requires their kinds to remain available to reconstruction.

11.2 Time without an imported continuum

The index i supplies an operational order. It need not presume a completed real-valued time axis. If a duration is measured, Δt_i may carry that finite observation; if only precedence is known, the link records precedence. The minimum condition is that the enacted sequence can be distinguished sufficiently to reproduce the calculation at the chosen resolution.

Accordingly, operational time is not an ornamental coordinate added after the proof. It is the order in which permissions are used. Classical notation often suppresses this order because a trained reader can reconstruct it. The rope restores it as part of the object under examination.

11.3 Reversibility and provenance

A link need not be reversible. When reversal is permitted, the inverse rule and its admissibility conditions must be stated. When a step depends on a definition, a theorem, a convention, or an observed agreement, that provenance belongs either to the link or to its accessible history. A mere line drawn between beads is insufficient because it cannot distinguish valid inference from visual proximity.

Chapter 12

The Nodal Rope

12.1 Definition

For a finite proof, calculation, sentence, or other symbolic performance $\mathcal{P}$, its operational unfolding at resolution α is the alternating finite sequence

$$\mathcal{R}_\alpha(\mathcal{P}) = (B_0, E_0, B_1, E_1, \ldots, E_{m-1}, B_m). \tag{12.1}$$

The same construction may be displayed more economically as

$$B_0 \xrightarrow{E_0} B_1 \xrightarrow{E_1} \cdots \xrightarrow{E_{m-1}} B_m. \tag{12.2}$$

This is the nodal rope. Its nodes are finite beads and its flexible intervals are finite operational links. The adjective *flexible* means that a later spatial realisation may bend the rope without changing its operational order or declared relations.

12.2 Why the rope is not the written line

A printed derivation may place two expressions on adjacent lines while omitting many enacted operations. Conversely, a single written expression may require a long evaluation path. The rope follows the calculation actually required under a declared reader model, not merely the typography. Its length is therefore relative to the available lattice of the agent who performs or reconstructs it.

For a reader who already possesses a lemma, one bead may call that lemma as an admissible unit. For another reader, the lemma must be unfolded into its own sub-rope. This relativity does not make the construction arbitrary. It makes the assumed compression boundary explicit.

12.3 Nested ropes and finite calls

If a bead invokes a previously validated finite procedure $\mathcal{Q}$, one may record

$$B_i \rightsquigarrow \mathcal{R}_\alpha(\mathcal{Q}) \tag{12.3}$$

together with the entry and return conditions. The call may remain compressed only when the reconstruction contract declares that $\mathcal{Q}$ is already available. A full audit recursively unfolds such calls until the chosen alphonic and consensual boundary is reached.

The result resembles a rope with finite knots or bundled intervals, but the metaphor must not conceal the order. Every bundle must have a finite route through its constituent operations. Higher-order matrices, function calls, and cited propositions are treated in precisely this way.

12.4 Equivalence of unfoldings

Two ropes need not be bead-for-bead identical to realise the same accepted calculation. Let

$$\mathcal{R}_\alpha(\mathcal{P}) \equiv_{\alpha,C} \mathcal{R}'_\alpha(\mathcal{P}) \tag{12.4}$$

mean that the two finite unfoldings are indistinguishable under the declared resolution α, consensus conditions C, terminal registration, and permitted reconstruction tests. This is not exact equality. It is a typed finite equivalence whose tests must accompany its use.

Chapter 13

The Finite Spherical Construction

13.1 A shell of finite beads

The target of the conjecture is not the completed point-set sphere $\{x \in \mathbb{R}^3 : \|x\| = R\}$ taken without qualification. It is a finite spherical construction composed of finitely many alphonic neighbourhoods. Write

$$\mathbb{S}_{\alpha,R} = \bigcup_{j=1}^{N} \mathcal{B}(s_j, \rho_j), \qquad \rho_j \geq \alpha, \tag{13.1}$$

subject to the finite radial band

$$R - \varepsilon_R \leq \|s_j - o\| \leq R + \varepsilon_R. \tag{13.2}$$

Here o is a chosen finite origin for this construction, R is a finite measured radius, and ε_R is the admitted radial error. The notation $\mathbb{S}_{\alpha,R}$ names the finite shell; it must not be confused with an exact classical surface.

13.2 Coverage and vacancies

The union in Equation (13.1) need not cover every point of an ideal sphere, because such a requirement would reintroduce the completed continuum. It must instead satisfy a declared finite coverage test. One possible test asks whether a finite sampling instrument, operating no more finely than α, encounters an admitted shell bead in every tested direction. Another may require only the set of locations used by the rope.

Vacant regions are therefore not automatically failures. Failure depends on the proposed function of the realisation. If the claim is that the entire finite shell participates, coverage must be measured. If the claim concerns only a routed rope, the occupied nodal and link neighbourhoods may suffice.

13.3 The origin as a constructional choice

The origin o organises the radial test but does not constitute a metaphysical origin of meaning. A different finite origin may permit a different spherical realisation. The conjecture concerns the existence of at least one admissible construction, followed by the measurement of its cost and uncertainty. Uniqueness is neither presumed nor presently required.

Chapter 14

Folding the Rope onto the Sphere

14.1 Placement is not yet realisation

Because a finite rope has finitely many beads, it is easy to assign each bead an arbitrary point on a drawn sphere. That weak placement proves almost nothing. A strong realisation must preserve operational order, distinguish required branch states, make uncertainty visible at the admitted scale, and support finite reconstruction.

Let

$$\Phi_\alpha : \mathcal{R}_\alpha(\mathcal{P}) \longrightarrow \mathbb{S}_{\alpha,R} \tag{14.1}$$

be a candidate folding. For each bead it assigns a finite shell neighbourhood, and for each link it assigns a finite route between the relevant neighbourhoods. The route may curve in three-dimensional space, but its flexibility cannot be used to erase the distinction between non-equivalent operations.

14.2 Finite admissibility conditions

A candidate folding is admitted only relative to stated tests. The minimal present tests require that every source bead and link has a finite image; that operational precedence can be recovered; that distinct states remain distinguishable whenever the calculation requires them; that crossings do not create undeclared connections; that branch choices and nested calls retain their provenance; and that all locations, radii, separations, and errors are given through finite registrations.

The crossing condition is particularly important. In three dimensions two rope segments can pass without intersecting, but a picture or finite measurement may fail to resolve their separation. The construction must therefore maintain either a measurable spatial clearance or an explicit non-connection symbol no finer than the accepted uncertainty.

14.3 The decoder

The reconstructive requirement may be written

$$D_\alpha(\Phi_\alpha(\mathcal{R}_\alpha(\mathcal{P}))) \equiv_{\alpha,C} \mathcal{R}_\alpha(\mathcal{P}), \tag{14.2}$$

where D_α is a finite decoding procedure. The decoder need not recover an imagined exact original beyond α. It must recover an operationally equivalent rope under the declared consensus and terminal tests.

This equation is the central safeguard against decorative geometry. Without a decoder, the sphere is merely an illustration. With a tested decoder, the spherical object becomes a finite alternative registration of the symbolic trajectory.

14.4 Order on a surface

Spatial nearness does not by itself encode sequence. The folding must therefore carry direction by an orientation convention, indexed links, local arrow states, colour classes, ordered neighbourhoods, or another finitely measurable device. No particular encoding is privileged at this stage. The experiment must determine which encoding remains reconstructible when the shell is crowded and uncertainty neighbourhoods overlap.

Chapter 15

The Spherical Nodal-Rope Conjecture

15.1 Weak form

For every finite symbolic trajectory $\mathcal{P}$, and for a stated alphonic resolution α, there exists a finite three-dimensional spherical construction on which the beads and links of $\mathcal{R}_\alpha(\mathcal{P})$ can be placed with finite error.

The weak form follows too readily if arbitrary enlargement, arbitrary auxiliary symbols, and mere placement are allowed. It is retained only as a preliminary geometric question. Its satisfaction would not by itself constitute an FSM proof.

15.2 Strong reconstructive form

For every finite symbolic trajectory $\mathcal{P}$ that is operationally admissible at resolution α, there exist finite parameters R, N, ε_R, a spherical construction $\mathbb{S}_{\alpha,R}$, a folding Φ_α, and a finite decoder D_α such that Equation (14.2) holds and all declared bead, link, order, distinction, provenance, and uncertainty conditions are preserved.

This is the substantive conjecture. It may be false. A counterexample may show that a required operational distinction cannot be maintained on the finite shell under the chosen constraints, that reconstruction demands information not contained in the realisation, or that the construction grows without a finite bound permitted by the experiment.

15.3 Bounded versions

The strongest universal statement should not obscure more tractable bounded questions. One may fix a maximum rope length m, a finite alphabet, a maximum nesting depth, a family of allowed operations, a minimum nodal separation, and a maximum shell radius. The experimental question is then whether every rope inside that class has an admissible spherical realisation and what resources the construction requires.

Such bounded results would be valuable even if the unrestricted conjecture failed. They would identify precisely which symbolic operations, uncertainty demands, or reconstruction conditions cause the failure.

Chapter 16

Spherical Realisation as One Form of FSM Proof

16.1 A constructive certificate

Within the present programme, an FSM realisation proof is not a replacement for a classical proof of a theorem. It is a finite certificate that a declared symbolic trajectory has been unfolded, spatially realised, and reconstructed without loss beyond the accepted boundary. A certificate may be represented by

$$\Pi_{\mathrm{FSM}}(mathcal P, \alpha, C) = (\mathcal{R}_\alpha, \mathbb{S}_{\alpha,R}, \Phi_\alpha, D_\alpha, \mathcal{T}_{\alpha,C}), \tag{16.1}$$

where $\mathcal{T}_{\alpha,C}$ is the finite record of admissibility and reconstruction tests.

The certificate proves only what its types state. It may certify the faithful finite realisation of a classically valid derivation; it does not supply that validity unless the inference rules and premises are themselves included and tested. It may also expose a gap in an otherwise accepted presentation by showing that a required transition cannot be reconstructed from the declared record.

16.2 Three non-equivalent achievements

It is essential to distinguish classical validity, operational completeness, and spherical realisability. Classical validity concerns whether a conclusion follows under an accepted formal system. Operational completeness concerns whether the finite sequence of required acts has been made available at the declared boundary. Spherical realisability concerns whether that sequence can be encoded on and decoded from a finite three-dimensional shell. Success in one category does not silently transfer to another.

16.3 Experimental measurement

A useful experiment records at least rope length, number and type of nested calls, shell bead count, occupied area or angular range under the finite instrument, minimum resolved separation, number of auxiliary routing symbols, decoding steps, and reconstruction errors. These quantities are not assumed to be universal invariants. They are measurements made under a declared apparatus and may change when the encoding or alphonic limit changes.

Part IV

Initial Tests and Boundaries

Chapter 17

Test I: Unfolding a Matrix Calculation

17.1 The compressed classical statement

Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}. \tag{17.1}$$

Classically their product is displayed as

$$AB = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}. \tag{17.2}$$

The compact equation is a completed relational object. It does not state the order in which an actual finite device reads entries, performs multiplications, stores intermediate values, adds results, and writes output cells.

17.2 One finite unfolding

For the first output entry, a possible rope segment is

$$\begin{aligned} &B_0 = (a, e; \text{read}), && B_1 = (ae; \text{multiply}), \\ &B_2 = (b, g; \text{read}), && B_3 = (bg; \text{multiply}), \\ &B_4 = (ae, bg; \text{retrieve}), && B_5 = (ae + bg; \text{add}), \\ &B_6 = ((AB)_{11} = ae + bg; \text{write}). && \end{aligned} \tag{17.3}$$

The punctuation in Equation (17.3) abbreviates full bead records of the form in Equation (10.1). Links carry the read, multiply, retrieve, add, and write permissions. The remaining three entries are unfolded by analogous but separately ordered segments.

17.3 What must survive the folding

A spherical realisation must preserve which row and column supplied each operand, which two products belong to each sum, when an intermediate result is retrieved, and which output location receives the result. A map that merely places the sixteen visible letters and four output expressions on a sphere is not sufficient. It has mapped the notation, not the calculation.

If the four output segments are evaluated sequentially, they form one rope. If a device evaluates them concurrently, the concurrency must itself be unfolded into finite scheduling, synchronisation, and merge operations before a single operational route is claimed. The matrix example therefore exposes the difference between a classical dependency structure and one enacted finite trajectory.

17.4 Provisional outcome

For any fixed finite matrix calculation, an elementary spherical placement appears constructible by choosing separated shell neighbourhoods and routing the finite links with recorded orientation. This is evidence only for a bounded case. The important result of the test is methodological: the object that is folded is Equation (17.3) and its continuations, not the rectangular matrix as a timeless block.

Chapter 18

Test II: Unfolding a Finite Branch Calculation

18.1 The classical curve

Consider the elliptic-curve form

$$y^2 = x(x-1)(x-\lambda), \qquad \lambda \notin \{0,1\}. \tag{18.1}$$

Classically, a complex value of x away from the branch locations admits two values of y. The completed curve, its two-sheeted description, and its compactification organise all such values at once.

18.2 A deliberately finite registration

Fix finite registered values $\widehat{\lambda}$ and $\widehat{x}_0$ with uncertainty neighbourhoods that remain distinguishable from the prohibited and branch states at the chosen α. A calculation of one continuation may be unfolded as

$$\widehat{x}_0 \longrightarrow \widehat{x}_0 - 1 \longrightarrow \widehat{x}_0 - \widehat{\lambda} \longrightarrow \widehat{p}_0 \longrightarrow \widehat{y}_0^{+} \longrightarrow b_0 = +, \tag{18.2}$$

where $\widehat{p}_0$ is the finitely calculated product and the superscript and branch bead record the selected square-root continuation. A later step to $\widehat{x}_1$ must carry enough information to determine whether the current branch continues, becomes indistinguishable within uncertainty, or requires a new rule.

The hats are reminders that these are finite registrations, not exact possession of arbitrary complex numbers. The calculation may use rational pairs, interval enclosures, floating-point states, or another finite representation, but the representation and its error propagation must be declared.

18.3 Branch proximity as a measured event

Suppose the uncertainty neighbourhood of $\widehat{x}_i$ approaches that of a classical branch value. The rope does not announce that it has reached an exact mathematical branch point unless its finite system permits that exact symbolic identification. It records instead whether the two candidate y-neighbourhoods remain resolvable under α. The operational event is therefore

$$\operatorname{sep}_\alpha(U(y_i^+), U(y_i^-)) \in \{\text{resolved}, \text{unresolved}\}, \tag{18.3}$$

together with the rule used after that registration.

18.4 What the test corrects

No classical sheet is converted into a spherical shell. The rope contains successive calculations, branch-state beads, and continuation links. Those finite elements may then be tested for a spherical realisation. The familiar four branch points of a common elliptic presentation do not become four privileged spherical nodes by declaration. They contribute operational events only when the finite trajectory encounters, distinguishes, or symbolically invokes their associated rules.

Chapter 19

Test III: Unfolding a Short Formal Proof

19.1 The written proof

Take the elementary identity

$$(a+b)^2 = a^2 + 2ab + b^2. \tag{19.1}$$

In a commutative ring this may be justified by expansion, distributivity, and collection of equal terms. Its familiarity makes it useful: the visible line is short, yet even this line relies on several permissions.

19.2 An operational path

One finite symbolic unfolding is

$$\begin{aligned} (a+b)^2 &\longrightarrow (a+b)(a+b) \\ &\longrightarrow a(a+b) + b(a+b) \\ &\longrightarrow aa + ab + ba + bb \\ &\longrightarrow a^2 + ab + ab + b^2 \\ &\longrightarrow a^2 + 2ab + b^2. \end{aligned} \tag{19.2}$$

Each arrow must name its permission: the definition of the square, left and right distributivity, associational conventions, commutativity for $ba = ab$, and the accepted abbreviation $ab+ab = 2ab$. If the ambient algebra does not permit one of these operations, the rope fails even though the final inscription remains visually familiar.

19.3 Semantic nodes inside formal notation

The symbol 2 in the final line is not merely another bead copied from the premise. It records a finite construction or accepted scalar action. Likewise, equality signs in the displayed trajectory may indicate formal identity, a permitted rewrite, or equality of denotation under the stipulated structure. A semantic uncertainty register would mark these roles, especially when a document changes them without warning.

19.4 Spherical test

The rope in Equation (19.2) is finite and can readily be routed on a finite shell if each rewriting step and its orientation are retained. The test becomes more informative when compressed subproofs are substituted for the named algebraic laws. The sphere then records not only the visible five-line sequence but the finite depth at which the reader is expected to reconstruct its permissions.

Chapter 20

Failure Conditions and Counterexamples

20.1 Failure by loss of order

A spatial arrangement fails when two or more distinct operational orders decode to the same unmarked route and the difference matters to the result. Increasing the sphere's size may repair this by providing separation, but only if the added size remains part of a finite declared construction. If no finite ordering code is permitted, placement alone cannot satisfy the strong conjecture.

20.2 Failure by collision

When uncertainty neighbourhoods overlap so that required beads or links cannot be distinguished, the realisation fails at the selected α. The appropriate response is not to pretend that mathematical points have no extent. One must coarsen the claim, enlarge the construction, change the encoding, or record the case as a counterexample under those parameters.

20.3 Failure by hidden decompression

A decoder may appear finite only because it calls an undeclared external theorem, dictionary, convention, or algorithm. This is hidden decompression. The certificate fails unless the called resource lies inside the stated consensus boundary or is recursively supplied as a finite rope.

20.4 Failure by changing the object

If the folding replaces a multi-step operation with a new symbol but the decoder cannot recover the replaced steps, it has mapped a different trajectory. Compression is allowed only when its code and reconstruction permission are included. Likewise, a graph that records dependency but not enacted order cannot be presented as the operational rope without an additional scheduling rule.

20.5 Failure of the universal conjecture

The universal statement would fail if there existed an admissible finite rope for which no finite spherical construction and decoder preserved the declared constraints. A useful counterexample must state those constraints carefully. Prohibiting all auxiliary labels, fixing a sphere too small for the number of resolvable beads, or demanding exact recovery below α may manufacture failure by definition; permitting unlimited undeclared coding may manufacture success just as easily.

Chapter 21

Discussion: Meaning, Compression, and Consensus

21.1 Words as dense nodes

Words become dense when many possible histories, referents, analogies, and continuations are routed through a small inscription. Greater density can support finer understanding when a reader possesses the required lattice of distinctions. It can also create semantic instability when different sheets of use are silently treated as one. The rope makes this density observable as a demand for branching, provenance, and reconstruction rather than treating it as a mysterious property of the word.

The term *pattern* illustrates the danger. It may name a detected regularity, a representation, a causal organisation, a statistical recurrence, or an observer's compression. A sequence of sentences can remain locally fluent while moving among these roles. The operational rope must place a bead or link at the transition, or else admit that the decoder depends upon unrecorded reader repair.

21.2 Measurement and the nod

The framework does not escape language by appealing to measurement. Measurements are registered, compared, named, and accepted through finite practices. Yet the fall of a gold bar upon a foot is not made harmless by semantic theory. The deformation, pain report, image, diagnosis, and shared gesture constrain what may responsibly be said. The words map imperfectly, but they do not float free of observation.

Consensus in Equations (10.1) and (11.1) is therefore neither infallibility nor majority decree. It is the finitely available agreement that permits a sign or operation to function for the present task. Disagreement, revision, and measurement error can themselves be added to the rope. This is how meaning can transfer sufficiently for coordinated action while remaining uncertain.

21.3 A semantic uncertainty appendix

A full semantic uncertainty appendix may be demanding, but a modest branch register is already possible. For each high-load term it can record the active role, nearby alternatives, the context that selects the role, the consequences of a mistaken selection, and the residual uncertainty. This does not calculate meaning as a completed quantity. It makes some of the decoder's previously hidden work inspectable.

The present monograph treats such an appendix as a future documentary convention, not as a solved technology. Its immediate value is diagnostic. If an author cannot state which role a central term is playing at a decisive transition, the proposed rope contains an unresolved link.

21.4 Relation to classical mathematics

Classical mathematics remains indispensable. It supplies stable compressed objects, powerful equivalence rules, and carefully developed constraints. The FSM proposal does not ask that every mathematician replace these objects with laborious traces. It asks a different question: when one seeks a finite measured account of a proof or calculation, what operational trajectory must be reconstructed beneath the accepted compression?

Elliptic curves are relevant because they provide a rich test of continuation, periodic identification, branching, and compactification. They do not certify the conjecture in advance. Matrices are relevant because their notation clearly compresses repeated operations. They do not prove that every symbolic object unfolds in the same way. Each classical object must be translated through its own finite operational rules before comparison within a common rope framework becomes legitimate.

Chapter 22

Conclusion at the Boundary

The argument has established a disciplined form of the original intuition. A finite symbolic trajectory is first reconstructed as a nodal rope: a finite ordered sequence of beads and operational links, each carrying the uncertainty, history, provenance, and consensual permissions needed at the selected alphonic boundary. Only this rope, and not the completed classical object from which it may have been obtained, is offered for folding onto a finite spherical construction in three dimensions.

The spherical claim has consequently divided into a weak placement statement and a strong reconstructive conjecture. The weak statement is geometrically easy and conceptually insufficient. The strong statement demands a finite decoder that recovers an operationally equivalent rope while preserving order, distinctions, branch states, nested calls, and uncertainty. An admissible spherical realisation would constitute one form of FSM proof: not proof of the classical theorem by geometry alone, but a constructive certificate that its declared finite symbolic trajectory survives the transformation.

The initial matrix, branch, and algebraic tests show how the programme can begin without presuming its result. They also clarify what would count against it. Collisions at the alphonic scale, unrecoverable ordering, hidden decompression, or a finite rope that resists every admissible spherical encoding would narrow or refute the conjecture. This vulnerability is part of the proposal's value.

The stopping point is deliberate. Projection layers, delay reconstructions, Takens-type embeddings, integer lattices placed in three dimensions, retinal surfaces, and trajectories of Lexils constrained by Nexils belong to the next movement of the work. Importing them here would again risk allowing a suggestive classical or computational picture to arrive before the operational object has been secured. The bead, the link, the rope, and the finite sphere must remain visible in their own right.

Appendix A

Nomenclature and Typed Uses

Term	Present meaning	Excluded transfer
Alphon	A finite resolution boundary below which the present construction does not claim a distinction.	An exact universal atom of matter, meaning, or space.
Lexil	A finite symbolic bead currently functioning chiefly as a lexical or developed linguistic unit.	A claim that words possess fixed context-free meanings.
Nexil	A finite symbolic bead or link currently functioning chiefly as an operational constraint or relation.	A claim that mathematical signs are intrinsically unambiguous.
Bead	A finite symbolic state with history, uncertainty, consensus, and continuation data.	A dimensionless mathematical point or bare printed character.
Link	A finitely registered operation connecting successive bead states.	Mere visual adjacency or an unexplained arrow.
Rope	The finite ordered operational unfolding of a symbolic performance.	The classical proof, formula, graph, surface, or matrix taken as already unfolded.
Finite sphere	A finite shell of alphonic neighbourhoods satisfying declared radial and coverage tests.	The completed exact set of all points at distance R from an origin.

Term	Present meaning	Excluded transfer
Sheet	In the operational translation, a recorded branch state and continuation rule.	A literal FSM shell imported from a Riemann-surface drawing.
Branch point	A classical exceptional location translated into a finite distinction or rule-change event when encountered or invoked.	An automatically privileged spherical node.
Closure	A typed condition such as periodic identification, topological compactification, algebraic stability, logical termination, or semantic sufficiency.	Permission to move silently between these senses.

Appendix B

Classical Object Unfolding Register

Classical object	Compressed permission	Finite unfolding requirement	Spherical test object
Matrix	Simultaneous rectangular access to indexed entries and operations.	Ordered reads, arithmetic operations, storage, retrieval, and writes, with scheduling if parallelism is claimed.	The resulting bead–link rope, not the rectangle.
Tensor or higher-order array	Nested indexed relations displayed as one object.	A finite traversal rule, index states, local operations, and finite calls to nested subprocedures.	A rope with declared nesting and return conditions.
Complex plane	Completed coordinate field supporting complex arithmetic.	Finite registered coordinate pairs and arithmetic with error propagation.	The trajectory of coordinate operations.
Period lattice	Equivalence under translations by integer combinations of periods.	Finite period records, integer-step construction, comparison, and reduction rule.	The sequence of translation and equivalence events.
Complex torus	Quotient and topological compactification of the plane by a lattice.	Finite representative choice, boundary crossing, and equivalence registration.	A rope carrying quotient permissions; not the torus itself.

Classical object	Compressed permission	Finite unfolding requirement	Spherical test object
Elliptic function	Globally defined doubly periodic meromorphic relation.	Finite evaluation, period reduction, exceptional-state handling, and uncertainty.	Evaluation and continuation rope.
Elliptic curve	Completed algebraic or geometric solution object.	Finite substitutions, products, root choices, equality tests, and continuation rules.	Calculation rope with branch-state beads.
Riemann surface sheet	Local branch made globally coherent in the classical construction.	Branch label, continuation state, change condition, and finite provenance.	Branch-state intervals of the rope.
Branch point	Location where the covering description changes its local distinction structure.	Finite test of distinguishability and an explicit continuation-rule event.	A node only if the trajectory operationally registers it.
Point at infinity	Compactifying or projective element in the classical theory.	Reciprocal, projective, limiting, or symbolic closure instruction enacted finitely.	The finite closure-rule segment.
Formal proof	Completed sequence licensed by an inference system, often with compressed lemmas.	Ordered rule applications, substitutions, premise access, calls, returns, and terminal test.	The operational proof rope.

Appendix C

Provisional Admissibility Record

For an experiment on a trajectory $\mathcal{P}$, the following record should accompany any claim of spherical realisation. The record is written here as prose obligations rather than a universal axiom system because each measurement apparatus may demand additional fields.

The source declaration states the finite inscription or event stream, the formal or linguistic environment, the assumed reader resources, the alphonic limit α, and the consensus boundary C. The unfolding declaration supplies the bead and link schemas, the ordering rule, all retained nested calls, the terminal condition, and the procedure by which two ropes are judged equivalent.

The spherical declaration states the finite origin, radius, radial error, shell-bead count, bead radii, coverage test, routing symbols, orientation code, and minimum measured clearance. The decoding declaration supplies the finite procedure D_α, the resources it may access, its stopping condition, and the observations compared with the source rope.

The result declaration records successes, unresolved distinctions, collisions, substitutions, auxiliary symbols, reconstruction discrepancies, and the exact scope of the conclusion. A failed experiment is retained with the same provenance. Its failure may identify a limitation of the encoding, the selected shell, the resolution, or the conjecture itself.

Appendix D

Correction Record

The decisive correction in this monograph concerns the status of classical sheets. A Riemann surface is not interpreted as a pre-existing finite spherical shell, and its branch points are not copied directly into the FSM construction as four alignment beads. The classical object is first unfolded into the finite sequence of calculations, branch registrations, continuation decisions, and closure rules actually required by the selected procedure.

Only after that unfolding is complete is the resulting flexible nodal rope tested for placement and reconstruction on a finite three-dimensional spherical shell. This ordering prevents an attractive classical visualisation from deciding the FSM result in advance. It also preserves the possibility that the conjecture fails, which is necessary if the spherical realisation is to function as an experimental form of proof rather than as a metaphor protected from measurement.

Appendix E

Trajectory Boundary Marker

This monograph ends immediately before the turn toward projection layers, delay coordinates, Takens-type reconstruction, discrete integer lattices in three dimensions, retinal projection, and the placement of Lexil trajectories in Nexil-constrained spaces. Those ideas are intentionally not integrated into the present argument. They require a subsequent volume in which each imported construction is subjected to the same classical-object, finite-operation, nodal-rope, and spherical-test sequence.

The precise return marker is recorded as

$$\boxed{\text{TM–FSM–2026-08-18: NODAL-ROPE / RETINAL-PROJECTION TURN}}.$$