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The Geofinite Halting Thesis

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Overview

The Halting Problem, introduced by :contentReference[oaicite:0]index= in 1936, is widely regarded as one of the foundational results of theoretical computer science. It establishes that no general algorithm can determine, for every possible program and input, whether that program will halt or run indefinitely. This result has traditionally been interpreted as a fundamental and unbreachable limit on computation.

In this paper, we present a Geofinitist reframing of the Halting Problem. Rather than contesting the internal validity of Turing's proof, we examine the assumptions under which the problem is posed and demonstrate that its undecidability arises from commitments that extend beyond finite, measurable reality. By re-grounding computation within finite resources, measurable processes, and auditable outcomes, we reformulate halting as a bounded, empirical question. This leads to the formulation of the

Geofinite Halting Thesis, in which halting is no longer an absolute property, but a resource-dependent, operationally decidable condition admitting uncertainty and abstention.

Introduction

The classical Halting Problem asks whether there exists a universal procedure capable of determining, for any program P and input x, whether $P(x)$ will eventually halt. Turing's answer, derived via diagonalization, shows that no such procedure can exist. The contradiction arises from constructing a program that inverts the prediction of the supposed decider when applied to itself.

This result is mathematically rigorous and remains unchallenged within its formal domain. However, its interpretation as a statement about all possible computation relies on implicit assumptions: that programs may be arbitrarily specified, that execution may proceed over unbounded time and memory, and that halting is a well-defined, exact property across this domain.

Geofinitism challenges these assumptions by asserting that all meaningful computation occurs within finite, measurable constraints. From this perspective, the classical Halting Problem is not false, but inadmissible as a description of real computational processes.

The Classical Construction

Let $H(P, x)$ be a hypothetical halting decider that returns `halt` if $P(x)$ halts and `loop` otherwise. Turing constructs a program $D(P)$ such that:

$$D(P) = \begin{cases} \text{loop} & \text{if } H(P, P) = \texttt{halt}, \\ \text{halt} & \text{if } H(P, P) = \texttt{loop}. \end{cases}$$

Evaluating $D(D)$ produces a contradiction: if H predicts halting, D loops; if it predicts looping, D halts. Therefore, no such universal decider H can exist.

This proof depends critically on self-reference, exact classification, and total domain coverage. These conditions, while mathematically admissible, extend beyond the constraints of finite computation.

A Geofinitist Reframing

Geofinitism replaces the demand for universal, exact classification with a framework grounded in finite measurement, bounded resources, and observational uncertainty. Computation is understood not as a static mapping from input to output, but as a trajectory through a measurable configuration space.

Let M denote a machine model with configuration space $\mathcal{C}$ and transition function $\Phi : \mathcal{C} \to \mathcal{C}$. For a program-input pair (P, x), define a measured trace:

$$m_t = (c_t, \varepsilon_t, P_{\text{obs}}), \quad c_{t+1} = \Phi(c_t),$$

where c_t is the configuration at time t, ε_t represents measurement uncertainty, and P_{obs} records observational provenance.

Budgeted Halting

Introduce finite resource bounds $B = (T_{\max}, S_{\max})$ for time and space. Define the budgeted stopping time:

$$\tau_B = \inf\{t \leq T_{\max} : c_t \in \mathcal{H}\},$$

where $\mathcal{H}$ is the halting set.

The Geofinite halting outcome is then given by:

$$\mathsf{H}_B(P, x) = \begin{cases} \text{HALT} & \text{if } \tau_B < \infty, \\ \text{NO\_HALT\_WITHIN\_B} & \text{if } \tau_B = \infty \text{ and progress tests fail}, \\ \text{UNDERDETERMINED} & \text{otherwise.} \end{cases}$$

This replaces binary classification with a three-valued outcome reflecting observable behavior within finite limits.

Measured Progress and Uncertainty

Define a progress signal over configurations:

$$Q_t = \text{novelty or change metric over } c_t,$$

and evaluate over a window $[t, t+w]$. If both state change and novelty fall below thresholds (θ_S, θ_Q), the system may be classified as non-halting within the given budget.

Uncertainty is explicitly modeled through ε_t, and outcomes are reported with confidence bands $\gamma \in \mathbb{M}$, reflecting measurement fidelity and observational limits.

Certification and Abstention

Halting admits a constructive certificate: the explicit observation of a terminal configuration. Non-halting, however, is generally uncertifiable and must be expressed relative to resource bounds unless supported by a proof certificate (e.g., a detected cycle or ranking function).

The inclusion of the UNDERDETERMINED state is central. It acknowledges that finite systems cannot resolve all cases and avoids forcing conclusions beyond observational evidence.

7. The Geofinite Halting Thesis

We now state the central result:

> **Geofinite Halting Thesis.** The classical Halting Problem arises from an inadmissible extension of finite computational processes into a domain requiring total, exact, and unbounded classification over arbitrary symbolic constructions. Within a finite, measurable framework, halting is not a universal property but a resource-bounded, empirically adjudicated outcome, admitting certified halting, bounded non-halting evidence, and irreducible underdetermination.

Undecidability is thus reframed not as a property of computation itself, but as a consequence of imposing total classification beyond the limits of finite measurement.

Discussion

The Geofinitist reframing does not invalidate Turing's proof; rather, it situates it within a broader epistemic context. The classical result remains correct under its assumptions, but those assumptions do not correspond to physically realizable computation.

By grounding reasoning in finite resources and measurable outcomes, the Halting Problem is transformed from a universal impossibility into a structured, operational procedure. This aligns more closely with engineering practice, where decisions are made under constraints, with explicit uncertainty and bounded guarantees.

More broadly, this reframing suggests a shift in how computational limits are interpreted. Rather than viewing undecidability as a barrier, it may be understood as a boundary condition that defines the scope of meaningful computation. Within that scope, robust and reliable reasoning remains possible.

Conclusion

The Halting Problem has long been regarded as a definitive limit on computation. Through the lens of Geofinitism, it becomes instead a reflection of the assumptions we bring to the concept of computation itself. By rejecting idealized infinities and embracing finite, measurable processes, we recover a practical and coherent framework in which halting is not undecidable, but conditionally and operationally resolvable.

In this way, the Geofinite Halting Thesis preserves the insight of Turing while dissolving its apparent paradox, revealing that the limits of computation are not barriers to understanding, but guides to a more grounded and realistic conception of it.