The Attralucian Essays:

Exploring the Finite

First Edition

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Geofinitism: Commitment, Admissibility, and Stabilization

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Chapter 1

Geofinitism: Commitment, Admissibility, and Stabilization

Overview

Across mathematics, logic, computation, and physics, a recurring pattern appears. Certain problems—often called paradoxes or foundational questions—resist resolution not because they are poorly posed, but because they rely on structures that extend beyond what can be constructed, measured, or stabilized.

This chapter introduces Geofinitism as a methodological framework designed to address this pattern. Rather than attempting to resolve such problems within their original assumptions, Geofinitism reframes them by altering the criteria under which statements are considered meaningful. The central shift is from idealized, limit-based reasoning toward finite, operational, and reproducible processes.

Three guiding commitments define this framework:

- **Commitment**: All reasoning is grounded in finite symbolic and physical processes.
- **Admissibility**: Statements must correspond to realizable procedures with bounded uncertainty.
- **Stabilization**: Meaning and truth arise through convergence and stability, not absolute evaluation.

These commitments form the arc through which classical problems will be revisited in the chapters that follow.

1.1 The Geofinitist Commitment

The Geofinitist position begins with a simple but far-reaching observation: all interaction with mathematics, language, and physical systems occurs through finite acts of construction and measurement.

Symbols are written, transmitted, and interpreted through finite media. Measurements are taken with finite precision. Computations are executed with bounded time and memory. Even when discussing infinite objects, these discussions occur through finite representations.

Geofinitism does not deny the usefulness of infinite structures. Instead, it treats them as *useful fictions*: tools that extend reasoning beyond immediate construction, but whose validity depends on their connection to finite procedures.

This leads to a reformulation of mathematical practice. Rather than asking whether a statement is true in an abstract, completed universe, Geofinitism asks:

> What can be constructed, measured, reproduced, and stabilized within finite bounds?

1.2 The Admissibility Principle

At the core of Geofinitism lies a single axiom governing meaningful statements.

> **Admissibility Principle (central axiom of Geofinitism):**
> A statement is admissible if and only if it can be instantiated as a finite, reproducible procedure with bounded uncertainty and declared provenance.

Each component of this principle is essential:

- **Finite**: The procedure must terminate within bounded resources.
- **Reproducible**: Independent implementations yield consistent outcomes within tolerance.
- **Bounded Uncertainty**: All outputs carry explicit error bounds.

- **Declared Provenance**: The method, assumptions, and context are recorded.

This principle does not invalidate classical mathematics. Instead, it defines a boundary between:

- formal statements (valid within symbolic systems), and
- operational statements (valid within measurable reality).

Geofinitism concerns itself with the latter.

1.3 Stabilization and Consensus

In classical logic, truth is treated as a static assignment. A statement is either true or false.

Geofinitism replaces this with a dynamic perspective. Meaning and truth emerge through iterative processes and are validated by stability.

Let $T^{(k)}$ denote the evaluation of a statement after k interpretive or computational steps. A statement is said to stabilize if:

$$|T^{(k+1)} - T^{(k)}| < \theta$$

for some tolerance θ over a finite window.

If stabilization occurs, the statement is assigned a value within the corresponding tolerance band. If stabilization fails, the system records indeterminacy rather than contradiction.

This leads to a notion of *consensus*: agreement across independent evaluations within declared uncertainty bounds. Truth is not imposed; it is achieved through convergence.

1.4 Historical Context: The Emergence of Limit Problems

The problems addressed in this work arise from a long trajectory in the development of mathematics and philosophy.

Ancient Origins

Zeno's paradoxes, formulated in the 5th century BCE, already expose tensions between motion and infinite subdivision. They demonstrate that reasoning about infinite processes can conflict with observed reality.

The Rise of Set Theory

In the late 19th century, Cantor's set theory introduced a rigorous treatment of infinity. This led to powerful results, but also to paradoxes such as Russell's paradox, revealing contradictions in unrestricted comprehension.

Measure and Decomposition

The early 20th century saw the development of measure theory alongside constructions such as the Banach–Tarski paradox. These results showed that, under certain assumptions, volume and structure could behave counterintuitively.

Logic and Truth

Self-reference led to paradoxes such as the Liar, prompting developments in formal logic, including Tarski's hierarchy and Kripke's partial semantics.

Independence and Undecidability

Gödel and Cohen demonstrated that certain statements, including the Continuum Hypothesis, cannot be decided within standard axiomatic systems. This revealed limits not only of computation, but of formal reasoning itself.

A Common Pattern

Despite their diversity, these problems share a common structure:

- reliance on infinite or unbounded constructions,
- assumption of exact, context-free predicates,
- collapse of dynamic processes into static definitions,

- absence of explicit measurement or uncertainty,
- extension of reasoning beyond operational limits.

Geofinitism interprets these features not as deep mysteries, but as signals that admissibility has been exceeded.

1.5 From Limits to Stability

Classical mathematics often asks:

> What happens in the limit?

Geofinitism reframes the question:

> What remains stable under finite construction, measurement, and perturbation?

This shift has several consequences:

- Paradoxes become indicators of inadmissible constructions.
- Infinite processes are replaced by finite stopping criteria.
- Exact identities are replaced by tolerance-based equivalence.
- Absolute size is replaced by scaling behavior.
- Truth becomes a property of stabilized evaluation.

The focus moves from completion to convergence.

1.6 The Structure of This Work

The chapters that follow apply the Geofinitist framework to a series of classical problems:

- Russell's Paradox
- The Banach–Tarski Paradox
- Zeno's Paradoxes
- The Liar Paradox
- The Continuum Hypothesis

Each case study follows a common structure:

1. identification of classical assumptions,
2. analysis of inadmissible elements,
3. reformulation in finite, measurable terms,
4. demonstration of stabilization or boundary conditions.

The goal is not to refute classical mathematics, but to reinterpret its results within a framework aligned with finite reality.

1.7 Interpretation

Geofinitism does not eliminate infinity. It relocates it.

Infinity becomes a guiding abstraction, not a governing ontology. Its role is to suggest directions of extension, while admissibility determines which constructions remain meaningful.

Under this view:

- mathematics becomes a study of structured symbolic processes,
- truth becomes a stabilized property of those processes,
- paradoxes become diagnostic tools,
- and meaning is grounded in finite interaction.

Conclusion

The problems explored in this work are not failures of reasoning. They are reflections of a deeper tension between idealized abstraction and finite realization.

Geofinitism resolves this tension by introducing a simple constraint: admissibility.

Once finite construction, measurement, and stabilization are enforced, the apparent contradictions dissolve, and a consistent operational framework emerges.

The chapters that follow develop this perspective in detail, showing how classical problems transform when viewed through the lens of finite, measurable reality.