

The Attralucian Essays:
Exploring the Finite



First Edition

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The Attralucian Essays



The Five Pillars of Geofinitism

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The Five Pillars of Geofinitism

The Admissibility Principle provides the governing criterion for meaningful statements. The Five Pillars of Geofinitism articulate the structural commitments that support this principle. Together, they form a quasi-axiomatic foundation for reasoning within finite, measurable systems.

Each pillar identifies a class of assumptions commonly embedded in classical reasoning and replaces it with an operationally grounded alternative.

Pillar I: Geometric Container Space

Commitment. All constructions, interpretations, and transformations occur within a finite, structured container space.

Formal Statement. Let M denote a bounded manifold of admissible states. Any object, symbol, or system X is represented as a trajectory:

$$X : t \mapsto X(t) \in M,$$

with t indexing finite construction or evaluation steps.

Interpretation. Objects are not static, context-free entities; they are paths through a structured space of possibilities. Infinite or unbounded constructions that cannot

be embedded into M are inadmissible.

Pillar II: Approximations and Measurements

Commitment. All quantities are measured with finite precision and must carry explicit uncertainty.

Formal Statement. Every quantity q is represented as a measured value:

$$q \in \mathbb{M} \equiv (v_q, \varepsilon_q, P_q),$$

where v_q is the nominal value, ε_q is the uncertainty bound, and P_q is the provenance.

Interpretation. Exact values are limiting abstractions. All admissible reasoning propagates uncertainty explicitly. Any argument relying on perfect precision is outside the Geofinitist domain.

Pillar III: Dynamic Flow of Symbols

Commitment. Meaning and structure arise through finite, layered transformations rather than instantaneous evaluation.

Formal Statement. Let $X^{(k)}$ denote the state of a system at layer or step k . Then:

$$X^{(k+1)} = F(X^{(k)}, \Delta r_k),$$

where Δr_k represents finite resource increments (time, memory, resolution).

Interpretation. Symbolic systems evolve across layers (e.g., syntax \rightarrow semantics \rightarrow interpretation). Classical “one-step” definitions collapse this structure; Geofinitism restores it as a finite cascade.

Pillar IV: Useful Fiction

Commitment. Abstract constructs (e.g., infinity, exact equality, absolute truth) are treated as useful fictions whose validity depends on operational stability.

Formal Statement. A construct C is admissible only if its induced procedures stabilize under finite evaluation:

$\exists \theta > 0$ such that $|C^{(k+1)} - C^{(k)}| < \theta$ over a finite window.

Interpretation. Infinite or idealized entities are retained as tools, not ontological commitments. Their legitimacy is determined by whether they produce stable, reproducible outcomes within finite bounds.

Pillar V: Finite Reality

Commitment. All processes operate under finite resource constraints, including time, space, energy, and representational capacity.

Formal Statement. There exist minimal quanta:

$$\delta t > 0, \quad \delta x > 0,$$

and finite budgets:

$$T_{\max}, \quad S_{\max}, \quad E_{\max},$$

such that all admissible procedures terminate within these bounds.

Interpretation. Infinite regress, unbounded subdivision, and unrestricted recursion are excluded. Every admissible construction must terminate or stabilize within declared limits.

Synthesis

The Five Pillars collectively enforce a single structural principle:

All admissible reasoning must be grounded in finite trajectories within a bounded space, with explicit uncertainty, layered transformation, and resource constraints.

Under this synthesis:

- static definitions are replaced by trajectories,
- exact values are replaced by measured quantities,
- instantaneous evaluation is replaced by finite iter-

ation,

- absolute abstractions are replaced by stability criteria,
- infinite processes are replaced by bounded procedures.

These pillars do not restrict mathematical creativity. Rather, they define the boundary within which symbolic constructions maintain operational meaning.

Role in This Work

Each of the problems examined in the following chapters can be understood as a violation of one or more pillars:

- Russell's Paradox violates bounded construction (Pillars I, V),
- Banach–Tarski violates measurability and finite decomposition (Pillars II, V),
- Zeno's Paradoxes violate finite resolution (Pillars II, V),
- The Liar Paradox violates stabilization (Pillar IV),
- The Continuum Hypothesis exceeds operational distinguishability (Pillars II, IV).

The Geofinitist method is therefore not problem-specific. It is structural: identify inadmissible assumptions, en-

force the five pillars, and observe the resulting stabilization or boundary condition.

Remark

The Five Pillars presented here form the philosophical and methodological foundation of Geofinitism. More formal logical developments, including extensions such as Aphonic Logic, will refine these principles into explicit symbolic systems. The present work operates at the level of foundational commitments, preparing the ground for those later constructions.