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Exploring the Finite

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The Liar Paradox: A Geofinitist Reinterpretation

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Overview

The Liar Paradox arises from self-referential truth attribution, producing an apparent contradiction: if a sentence asserts its own falsity, it can be neither consistently true nor false under classical bivalence.

This paper does not claim to resolve the paradox in a metaphysical sense. Instead, it reinterprets it within a Geofinitist framework, where truth is treated as a finite, measured, and context-dependent process. Under these conditions, the Liar is not contradictory but *indeterminate*, reflecting a failure of admissible stabilization.

Classical Formulation

Consider the sentence:

$$L: \quad \text{“This sentence is false.”}$$

Under classical bivalence:

- If L is true, then it is false.
- If L is false, then it is true.

This produces a contradiction. Classical responses in-

clude:

- hierarchical separation of truth predicates,
- partial truth assignments,
- fixed-point semantics.

Source of the Paradox

The contradiction arises from:

- unrestricted self-reference,
- global truth evaluation,
- strict bivalence,
- absence of evaluation dynamics,
- lack of resource or stability constraints.

These allow a sentence to demand a fixed truth value where none can stabilize.

Geofinitist Principles Applied

1. Truth as Trajectory

Truth is not a static assignment but a trajectory through interpretive states, evolving across context and evaluation steps.

2. Measured Truth

Truth is represented as a measured quantity:

$$T(\sigma) = (v_\sigma, \varepsilon_\sigma), \quad v_\sigma \in [0, 1].$$

3. Layered Interpretation

Evaluation proceeds through finite layers (syntax, semantics, pragmatics), each updating the truth assignment.

4. Admissibility

A sentence is admissible if its truth value stabilizes under bounded evaluation.

5. Finite Evaluation

All truth assignments are computed within finite steps and bounded resources.

Geofinitist Reformulation

Let $\mathcal{S}$ be a set of sentences. Define a measured truth valuation:

$$T : \mathcal{S} \to \mathbb{M}, \qquad T(\sigma) = (v_\sigma, \varepsilon_\sigma, P_\sigma).$$

Define a three-zone decision rule:

$$\text{truth}(\sigma) = \begin{cases} \textsc{true} & v_\sigma \geq 1 - \delta, \\ \textsc{false} & v_\sigma \leq \delta, \\ \textsc{indeterminate} & \text{otherwise.} \end{cases}$$

Self-Reference and Dynamics

Define an update operator:

$$T^{(k+1)} = \mathsf{U}(T^{(k)}).$$

For the Liar:

$$v_L^{(k+1)} \approx 1 - v_L^{(k)}.$$

This produces oscillation or convergence to:

$$v_L \approx \tfrac{1}{2}.$$

Thus:

$$T(L) = \left(\tfrac{1}{2}, \varepsilon_L\right), \quad \text{truth}(L) = \textsc{indeterminate}.$$

Stability Criterion

A sentence is classified only if:

$$|T^{(k+1)}(\sigma) - T^{(k)}(\sigma)| < \theta$$

over a finite window.

If no stabilization occurs, the sentence is assigned to the indeterminate band.

Inference Discipline

Inference is restricted:

- Only TRUE statements support deduction,
- Indeterminate statements do not propagate conclusions,
- Contradictions do not produce explosion.

Interpretation

Under this framework:

- Truth is a measured, context-dependent process,
- Self-reference is handled through dynamic evaluation,
- Contradiction is replaced by instability,
- Indeterminacy is a valid outcome.

Conclusion

The Liar Paradox does not demonstrate inconsistency in logic, but a failure of stabilization under unrestricted self-

reference. Classical systems address this through stratification or partial semantics.

Geofinitism reframes the issue operationally: truth must be computed, measured, and stabilized. Where stabilization fails, indeterminacy is reported. The paradox is thus reclassified as a boundary condition on admissible truth assignment.

Context. The Liar Paradox is treated as a failure of stable truth assignment under self-reference.

Measured Truth.

$$T(\sigma) = (v_\sigma, \varepsilon_\sigma, P_\sigma)$$

Decision Rule.

$$\text{TRUE},\ \text{FALSE},\ \text{INDETERMINATE}$$

Update Operator.

$$T^{(k+1)} = \mathsf{U}(T^{(k)})$$

Liar Constraint.

$$v_L \approx 1 - v_L \Rightarrow v_L \approx \tfrac{1}{2}$$

Stability. If no fixed point emerges, assign INDETERMINATE.

Inference. No inference from indeterminate statements.

Collapse Note. Bivalence is recovered when self-reference is absent.

Interpretation. The paradox reflects instability, not contradiction.