The Attralucian Essays: Exploring the Finite

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Russell's Paradox: A Geofinitist Reinterpretation

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Overview

Russell's paradox arises from unrestricted set formation, exposing a contradiction in naive comprehension. Rather than treating this as a failure of logic itself, this paper interprets the paradox as a boundary violation: a symbolic construction that exceeds admissible limits.

Within a Geofinitist framework, sets are not static totalities but finite constructions with provenance, bounded scope, and measurable criteria. Under these constraints, Russell's paradox is reclassified—not as a contradiction—but as an inadmissible construction.

Classical Formulation

Naive set theory permits unrestricted comprehension:

$$R = \{ S \mid S \notin S \}.$$

The question $R \in R$ yields:

$$R \in R \iff R \notin R,$$

a contradiction first identified by Russell (1901).

This result undermined Frege's logical program and led to the development of restricted systems such as type theory and axiomatic set theory (e.g. ZF/ZFC), where comprehension is constrained.

Source of the Paradox

The contradiction arises from three simultaneous assumptions:

- **Unrestricted comprehension:** any predicate defines a set.
- **Global membership:** membership is a total, context-free relation.
- **Self-reference without constraint:** sets may quantify over themselves without restriction.

Together, these allow the construction of a self-referential object that cannot stabilize.

Geofinitist Principles Applied

1. Finite Construction

Sets arise through explicit generative procedures with bounded resources. A set must be constructible within a finite process.

2. Measured Membership

Membership is not an absolute predicate but a context-dependent determination:

$S \in T \quad$ is evaluated within a finite procedure and tolerance.

3. Layered Formation

Collections are built across levels (elements, sets, collections of sets). Self-reference requires explicit stratification.

4. Admissibility

A symbolic construction is admissible only if it:

- arises from a finite generative process,
- is evaluable within bounded resources,
- stabilizes under finite perturbations.

5. Finite Scope

All constructions operate within declared bounds on size, depth, or complexity.

Geofinitist Reformulation

Replace unrestricted comprehension with bounded construction:

$$R' = \{\, S \mid S \notin S \ \land\ N(S) \le N_{\max} \,\},$$

where $N(S)$ measures construction depth or size.

Membership is evaluated procedurally:

Decide $S \in R'$ by a finite algorithm with bounded depth.

If evaluation requires exceeding $N_{\max}$, the construction is INADMISSIBLE.

Where the Paradox Breaks

The Russell construction fails admissibility:

- It requires evaluating membership over an unbounded totality,
- It introduces uncontrolled self-reference,
- It cannot be resolved within finite resources.

Thus the contradiction is not realized; instead, the construction is rejected as undefined within the admissible domain.

Interpretation

Under this framework:

- Sets are finite constructions, not totalities,
- Membership is a procedure, not an oracle,
- Self-reference is allowed only when stratified and bounded,
- Paradox signals inadmissibility, not inconsistency.

Conclusion

Russell's paradox does not indicate a failure of logic but a violation of constructional constraints. Classical systems such as ZF/ZFC resolve this by restricting comprehension.

Geofinitism reframes the issue more generally: symbolic constructions must be finite, measurable, and stabilizable. Under these conditions, paradoxical objects are excluded not by axiomatic prohibition, but by failure of admissibility.

Context. Russell's paradox arises from unrestricted comprehension and self-reference. We model sets as finite constructions with bounded procedures and provenance.

Measured Construction. A set is defined by a generative procedure:

$$S = \mathsf{Gen}(\theta, B),$$

where θ are rules and B is a finite resource bound.

Membership. Membership is a computed relation:

$$S \in T \iff \mathsf{Eval}(S, T) = (\text{true}, \varepsilon, P),$$

evaluated within finite depth and uncertainty.

Admissibility. A set S is admissible if:

- Gen terminates within B,
- Eval is well-defined,
- recursive dependencies are bounded.

Russell Construction.

$$R = \{S \mid S \notin S\}$$

is inadmissible because evaluation of $S \in S$ requires unbounded recursion.

Decision. If evaluation exceeds bounds, return UNDEFINED rather than a truth value.

Collapse Note. When bounds are removed, the construction reduces to the classical paradox.