The Attralucian Essays: Exploring the Finite

First Edition

 Typeset in LaTeX

The Attralucian Essays

The Church–Turing Thesis: A Geofinitist Reinterpretation

Kevin R. Haylett

The Church–Turing Thesis: A Geofinitist Reinterpretation

Overview

The Church–Turing Thesis (CTT) is one of the foundational claims of computer science, asserting that any function that is effectively calculable can be computed by a Turing machine. Traditionally, this thesis is understood within an abstract, asymptotic framework that deliberately idealises computation: infinite tapes, perfect symbols, and unbounded precision.

These abstractions have proven extraordinarily powerful. However, when computation is interpreted as a *physical process*, additional considerations arise: finite resources, measurement uncertainty, and reproducibility. This paper does not reject the Church–Turing Thesis. Rather, it reinterprets it within a Geofinitist framework, where all quantities are finite, measured, and accompanied by uncertainty.

Within this perspective, computability becomes an *operational and auditable property*: a claim about reproducible transformations under declared resource budgets, tolerances, and provenance.

From Classical to Physical Computation

The Church–Turing Thesis has multiple interpretations:

- **Classical CTT:** Every effectively calculable function is computable by a Turing machine.
- **Extended CTT:** Any reasonable model of computation can be simulated efficiently (typically with polynomial overhead) by a Turing machine.
- **Physical CTT:** Any physically realizable process can be simulated by a Turing machine.

These formulations are often conflated but differ in scope. The classical thesis concerns *possibility*, not efficiency. The extended thesis introduces efficiency assumptions. The physical thesis attempts to connect formal computation with physical reality.

The Geofinitist perspective primarily addresses the latter two: it asks how computability claims behave when grounded in finite, measurable systems.

A Measured View of Computation

Consider a simple example: computing $f(x) = x^2$. A human with pencil and paper, a classical digital computer, and a quantum device may all implement this transformation. Classical theory asserts their equivalence at the level of computability.

Geofinitism refines this equivalence: these systems produce *trajectories* through a space of finite resources—time, memory, energy, and precision. Computation is not a static mapping, but a *measured process* unfolding under constraints.

Geofinitist Pillars Applied to Computation

Computation as a Trajectory

Classically, computation is represented as a discrete sequence of symbolic steps. In a Geofinitist view, it is a trajectory through a resource manifold M, incorporating:

- temporal progression,
- memory allocation,
- energy expenditure,
- precision constraints.

Computability becomes a property of accessible paths within M.

Measurement and Uncertainty

Idealised computation assumes perfect symbols and exact transitions. Physical systems exhibit noise, latency, and variability. Computation must therefore be expressed as a measured quantity:

$$T \pm \varepsilon_T, \quad S \pm \varepsilon_S, \quad y \pm \varepsilon_y,$$

where uncertainties arise from both hardware and instance variability.

Layered Computational Structure

Computation occurs across multiple interacting layers:

- physical substrate,
- microarchitecture,
- algorithmic representation.

These layers introduce constraints and transformations that are collapsed in classical formulations but are explicit in measured systems.

Computability as Useful Fiction

The classical notion of "effective calculability" is not directly measurable. It functions as a limiting concept that guides formal reasoning. Within Geofinitism, it is treated as a *useful fiction*, whose operational meaning must be grounded in measurable procedures.

Finite Constraints

All physical computation is bounded by:

- finite time,
- finite memory,
- finite precision,
- finite energy.

Computability claims must therefore be interpreted within finite regimes rather than infinite limits.

A Geofinitist Formalisation

Let $\Sigma^\star$ denote finite strings over an alphabet. A physical device D with parameters θ induces a measured procedure:

$$\mathsf{Proc}_{D,\theta}(x) = \Big(y,\ \varepsilon_y,\ P_D;\ T,\ \varepsilon_T;\ S,\ \varepsilon_S\Big),$$

where P_D encodes provenance (hardware, calibration, noise model).

Operational Computability

A partial function $f : \Sigma^\star \rightharpoonup \Sigma^\star$ is (τ, δ)-*computable* by D on domain $\mathcal{X}$ if:

$$\forall x \in \mathcal{X} : \quad \Pr\Big[d_{\mathbb{M}}\big(\mathsf{Proc}_{D,\theta}(x), f(x)\big) \leq \tau\Big] \geq 1 - \delta,$$

within declared resource budgets.

Here $d_{\mathbb{M}}$ measures deviation between produced and target outputs within a finite symbolic representation.

Emulation and Universality

A universal machine U emulates D if there exist encoding and program mappings such that:

$$\forall x : \quad \Pr\Big[d_{\mathbb{M}}\big(U(p_D, E(x)), \mathsf{Proc}_{D,\theta}(x)\big) \le \tau\Big] \ge 1 - \delta.$$

Unlike classical formulations, resource overhead is *measured and reported*, not assumed polynomial. This accommodates distinctions between classical, extended, and physical interpretations of the thesis.

Geofinitist Church–Turing Thesis

We propose:

$$\boxed{\mathbf{CTT}_{\mathbb{M}} : \text{ All admissible physical procedures are emulable by a univer}}$$

Admissibility Criteria

A device is admissible if it satisfies:

- finite energy per step,

- bounded precision in input and output,
- reproducibility within tolerance,
- local causal evolution.

Devices requiring infinite precision, unbounded energy, or non-reproducible behaviour are classified as INADMISSIBLE, rather than "super-Turing."

Models of Computation

- **Randomised:** Probabilistic behaviour captured via measured distributions.
- **Analog:** Requires discretisation under finite resolution constraints.
- **Quantum:** Emulation is defined to finite tolerance; efficiency depends on resource scaling and is not assumed polynomial.
- **Oracle Models:** External resources are treated as provenance, not intrinsic computational power.

Interpretation

Within this framework, the Church–Turing Thesis is not a statement about abstract equivalence alone. It becomes an *empirical universality principle*:

- computability is reproducibility,
- equivalence is emulation within tolerance,

- universality is experimentally corroborated.

Conclusion

The classical Church–Turing Thesis remains a powerful abstraction. Geofinitism does not replace it, but *finitises* it. Computability is recast as a measurable property of physical systems, grounded in finite resources, uncertainty, and reproducibility.

This shift preserves the insight of the original thesis while aligning it with the realities of computation as it is practiced: not in infinite idealisations, but in finite, observable processes.