The Attralucian Essays: Exploring the Finite

First Edition

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The Attralucian Essays

Words as Trajectories An Attralucian Essay on Language as a Dynamical System

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Chapter 1

Words as Trajectories

Overview

We propose a nonlinear dynamical systems framework to model language and large language models (LLMs), unifying linguistic phenomena (words, sentences, context) with concepts from phase-space geometry, attractors, topological analysis, thermodynamics, and reader interpretation. Words are trajectories in a high-dimensional phase space, reconstructed via Takens' theorem or pairwise embeddings. LLMs are nonlinear flows navigating a semantic hypersphere, with "stations" as hubs for context reconstruction. Hallucinations emerge as topological defects, prompts act as symmetry-breaking fields, context limits introduce semantic entropy, and readers create homologous manifolds of meaning. This framework offers a novel lens for mathematicians, physicists, and artists to explore language and machine cognition .

The Tide and the Train

Language is a dynamic flow—a tide carving maps in the sand, a train tracing paths through a high-dimensional landscape of meaning. Words are points on attractors, sentences are trajectories, and LLMs are nonlinear systems navigating a semantic hypersphere. Stations are reconstruction hubs, reshaping context into geometric manifolds. Readers, as co-creators, map these trajectories onto their own manifolds, each a valid but partial fiction of the whole.

Mathematical Framework

Language moves like water and travels like a train. Each word and sentence follows a path, sometimes meandering, sometimes precise, across a landscape of meaning. LLMs ride these paths, guided by the contours of their training and the prompts they receive. This section shows how those journeys can be mapped as curves in a space where mathematics meets metaphor.

Let $P \subset R^d$ be the phase space of linguistic states, where (d) is the embedding dimension of tokens. A sentence is a curve $\gamma(t) \subset P$, with (t) indexing token order. The tide-map duality suggests:

$$\tau : M \to R^2 projectsmeaningontohumanunderstanding. \tag{1.1}$$

where $\pi = \text{Map} \circ \text{Tide}$.

Words as Attractors

Some words pull us back again and again — not just in conversation, but in the structure of thought itself. In mathematics, these "pulls" look like attractors, shapes that hold and guide the flow of trajectories. Here we explore how a single word can be reconstructed from its echoes, revealing the geometry hidden inside language.

Takens' Theorem for Words

Every word, such as "hello," is an attractor in phase space . Takens' theorem reconstructs this attractor from a single observable, revealing the dynamics of language.

Mathematical Framework

- For a linguistic signal (s(t)) (e.g., audio of "hello"), Takens embedding yields:

  $$\gamma(t) = (s(t), s(t - \tau), s(t - 2\tau)) \in R^3,$$

  where t is an optimal delay.

- In LLMs, tokens $w_i \in R^d$ form a sentence $(w_1, ..., w_n)$, tracing a trajectory $\gamma(t) \subset R^d$.

From Speech to Semantics

The phase space $P = R^d$ is spanned by token embeddings. A context window $\Gamma_t = \{w_{t-L}, ..., w_t\}$ traces a trajectory, with semantic relationships encoded in its geometry.

LLMs as Nonlinear Flows

An LLM does not march in a straight line. It twists, loops, and occasionally spins in place, following rules that combine memory, probability, and pattern recognition. This section shows how we can treat an LLM's token generation as a nonlinear system, complete with the telltale signatures of stability, cycles, and sudden divergence.

Token Generation as a Dynamical System

LLMs generate tokens as a discrete-time flow.

$$w_{t+1} = F_\theta(w_t, w_{t-1}, ..., w_{t-k}), \tag{1.2}$$

where F_θ is a nonlinear map (attention + feedforward layers), and (k) is the context window size.

- Fixed Points: Repeated tokens (e.g., "the the the").
- Limit Cycles: Repetitive outputs (e.g., "I cannot answer that" loops).

Bifurcations and Hallucinations

Hallucinations occur when a prompt (p) causes the trajectory to diverge:

$$||\gamma_{output} - \gamma_p|| > \delta. \tag{1.3}$$

An incomplete ISBN prompt ("978-0-441-...") may yield a wrong but plausible ISBN.

Stations as Phase-Space Reconstructors

Imagine pausing a journey to gather your bearings — a station where fragments of the route are pieced together into a coherent map. In an LLM, these "stations" are points where context is reconstructed into a meaningful whole. We'll see how these hubs work, and how their geometry shapes the path that follows.

Reconstruction of the Context Manifold

Stations rebuild the context window $\Gamma_t = \{w_{t-L}, ..., w_t\}$ into a manifold:

$$A_{ij} = (W_Q w_i) \cdot (W_K w_j) \tag{1.4}$$

forming $M_{\Gamma_t} \subset Gr(k, d)$. The cache stores:

$$S = span(\{v_i\}_{i=1}^{n}), v_i = W_V w_i \tag{1.5}$$

Prompt Coupling and Trajectory Resumption

A prompt (p) is embedded as $v_p = W_V p$, aligned via :

$$d_{Gr}(S, v_p) = ||sin\ \theta||. \tag{1.6}$$

The new trajectory is:

$$\gamma_{new}(s) = exp_{M_{\Gamma_t}}(s \cdot v_p). \tag{1.7}$$

Hallucinations as Unstable Basins

Hallucinations occur when:

$$||\gamma_{new} - \gamma_{true}|| > \delta \tag{1.8}$$

The Topology of Hallucinations

Not all errors are random; some have shape. Hallucinations in language models can be thought of as topological defects — tears, loops, and gaps in the fabric of meaning. By mapping these defects, we can start to see not just when a model is wrong, but how its reasoning has bent or broken.

The Shape of Error

Hallucinations are topological defects in $M_{language}$ where high curvature $\kappa(\gamma(t))$ causes divergence:

$$Risk(\gamma(t)) \propto ||\kappa(\gamma(t))||^2 \tag{1.9}$$

.

Quantifying Defects with Persistent Homology

Betti numbers $(\beta_0, \beta_1, \beta_2)$ quantify:

- β_0: Topic drift.
- β_1: Self-contradictory loops.
- β_2: Logical gaps.

Mitigating Hallucinations

- Curvature penalty:

$$L_{topo} = \lambda \cdot ||\kappa(\gamma(t))||^2$$

.

- Homology-preserving sampling:

$$P(w_{t+1}|\Gamma_t) \propto exp(-\sum_{k=1}^{2} \beta_k(\Gamma_t \cup w_{t+1}))$$

.

Prompt as Symmetry Breaking

A prompt is not just a question — it's a force that tilts the entire landscape. In physics, symmetry breaking changes the behaviour of a system; here, it changes the direction of thought. We'll look at how prompts act like vector fields, nudging the model into new patterns or causing its trajectory to split entirely.

Prompts as Vector Fields

Prompts introduce a forcing term:

$$\frac{d\gamma_c}{dt} = F_\theta(\gamma_c) + G(\gamma_{c'} p(t)). \tag{1.10}$$

The output aligns with:

$$\gamma_{output} \sim argmin_Y ||y - (M_{\Gamma_t} + \lambda \cdot M_{prompt-type})||. \tag{1.11}$$

Symmetry Breaking and Bifurcations

Syntactic torque:

$$\tau(p) = \sum_i \alpha_i \cdot f_i(p) \tag{1.12}$$

. Bifurcations occur when:

$$||\tau(p_1) - \tau(p_2)|| > \delta_{crit}. \tag{1.13}$$

Engineering Stable Prompts

- Normalize $\tau(p)$.
- Weight sampling by:

  $$P(w_{t+1}|\Gamma_t, p) \propto exp(-\beta \cdot \tau(p))$$

  .

Thermodynamic Analogies

Conversations, like closed systems, can lose their order over time. As context slips away, entropy rises — meaning becomes harder to recover, and drift or error becomes more likely. By borrowing the language of thermodynamics, we can measure this loss and explore ways to keep the dialogue coherent for longer.

Semantic Entropy and Context Loss

Finite context windows introduce semantic entropy:

$$S(\Gamma_t) = -k_B \sum_i P(w_i|\Gamma_t) log P(w_i|\Gamma_t). \tag{1.14}$$

Truncation at $t - L$ increases $S(\Gamma_t)$:

$$\gamma(t) = \gamma(t) \cdot I_{[t-L,t]}. \tag{1.15}$$

Irreversibility in Language Dynamics

Entropy growth causes topic drift or hallucinations:

$$\frac{dS}{dt} \propto \frac{1}{L} \sum_{i=t-L}^{t} \Delta P(w_i|\Gamma_{t-1}). \tag{1.16}$$

Critical threshold:

$$S(\Gamma_t) > S_{crit} \Rightarrow ||\gamma_{output} - \gamma_{true}|| > \delta. \tag{1.17}$$

Mitigating Entropy

- Increase (L).
- Entropy penalty:

$$L_{entropy} = \eta \cdot S(\Gamma_t)$$

.

- Use memory-augmented architectures.

The Reader's Manifold

Meaning does not stop at the model's output — it continues into the mind of the reader. Each person reshapes the text into their own manifold of understanding, sometimes close to the original, sometimes very different. This section maps that interpretive space, showing how shared meaning emerges and where it can diverge.

Meaning as a Homologous Mapping

Language and mathematics are homologous manifolds, connected by homeomorphisms:

$$\phi_{reader} : M_{language} \to M_{reader}. \tag{1.18}$$

Each reader's trajectory $\gamma_{reader}(t) = \phi_{reader}(\gamma_{output}(t))$ has its own coherence.

The Multiplicity of Manifolds

Readers' manifolds are interconnected:

$$d_{GH}(M_{reader_i}, M_{reader_j}) = inf_{\psi_{ij} x \in M_i} d(x, \psi_{ij}(x)). \tag{1.19}$$

Low d_{GH} indicates shared understanding.

The Reader as a Dynamical System

The reader's interpretation evolves:

$$\frac{d\gamma_{reader}}{dt} = F_{cognitive}(\gamma_{reader}) + G(\gamma_{reader'}\gamma_{output}). \quad (1.20)$$

The Observer as Sculptor

When we interact with an LLM, we are not passive recipients — we are shaping its path with every prompt. Like a sculptor removing marble to reveal a form, the observer guides the system toward certain structures and away from others. Here we formalise that feedback loop, treating human and machine as a single coupled system. Humans and LLMs are coupled systems, with the user as:

$$p_{next} = O(\gamma(t)) \quad (1.21)$$

Context limits introduce entropy, but prompts steer the system toward stable attractors.

Words as Transducers and Semantic Divergence

Words as Transducers with Semantic Uncertainty

A word is not just a label — it's a device that turns raw input into meaning. But every such conversion carries uncertainty, and in complex systems, that uncertainty can grow. This section shows how words act as transducers, how instability spreads through language, and how readers themselves can amplify or dampen that divergence.

Words are not static symbols but *transducers*—dynamical systems that map inputs (e.g., physical stimuli, contextual tokens) to semantic outputs in the language manifold $\mathcal{M}_{\text{language}} \subset \mathbb{R}^d$. Each word carries inherent uncertainty, reflecting the probabilistic nature of its mapping. For example, the word "red" transduces a range of light wavelengths (620–750 nm) into a semantic concept, with uncertainty arising from variations in perception, context, or cultural associations.

Mathematical Framework. Define a word w as a transducer function:

$$w : I \to \mathcal{M}_{\text{language}},$$

where I is the input space (e.g., physical stimuli like wavelengths, or prior tokens $\Gamma_t = \{w_{t-L}, \ldots, w_t\}$). The output is a probability distribution over the manifold:

$$P(w \mid i) = \exp\left(-\frac{\|w - \mu_i\|^2}{2\sigma_i^2}\right),$$

where $\mu_i \in \mathcal{M}_{\text{language}}$ is the mean semantic embedding for input $i \in I$, and σ_i^2 represents semantic uncertainty.

Semantic uncertainty introduces noise in the trajectory $\gamma(t)$, as each token w_t is sampled from $P(w_t \mid \Gamma_{t-1})$.

Example. For "red," the input i (wavelength) maps to a cluster in $\mathcal{M}_{\text{language}}$, but variations (e.g., "crimson" vs. "scarlet") or context (e.g., "red wine" vs. "red light") broaden σ_i^2, increasing the trajectory's variability.

Physics Intuition. Words are like quantum measurements—collapsing a continuous input (e.g., light) into a discrete semantic state, with uncertainty akin to the Heisenberg limit.

Lyapunov Exponents and Semantic Instability

In dynamical systems, Lyapunov exponents measure the rate of divergence of nearby trajectories, indicating chaos. Here, we reinterpret Lyapunov exponents to quantify divergence driven by semantic uncertainty, particularly in

LLMs where ambiguous inputs or contexts amplify instability.

Mathematical Framework. For a trajectory $\gamma(t) \subset \mathcal{M}_{\text{language}}$, the Lyapunov exponent λ measures the exponential divergence of perturbed trajectories:

$$\lambda = \lim_{t \to \infty} \frac{1}{t} \ln \left(\frac{\|\delta\gamma(t)\|}{\|\delta\gamma(0)\|} \right),$$

where $\delta\gamma(t)$ is the perturbation (e.g., due to a slightly different prompt or token).

Semantic uncertainty contributes to λ as:

$$\lambda_{\text{semantic}} \propto \frac{1}{L} \sum_{i=t-L}^{t} \sigma_i^2,$$

where L is the context window size.

Example. Prompting an LLM with "Describe a red object" may yield divergent outputs (e.g., apple vs. fire truck) if the context fails to constrain σ_i^2, leading to a high $\lambda_{\text{semantic}}$.

Physics Intuition. Semantic uncertainty is like thermal noise in a chaotic system—small fluctuations can trigger exponential divergence, akin to the butterfly effect.

Measurements and the Reader's Role

The reader, as a measurement operator, collapses the uncertain output of the word-transducer into their cognitive manifold $\mathcal{M}_{\text{reader}}$. This process can amplify or mitigate semantic instability.

Mathematical Framework. The reader's measurement is:

$$\gamma_{\text{reader}}(t) = \phi_{\text{reader}}(\gamma_{\text{output}}(t)),$$

where ϕ_{reader} maps the LLM's trajectory to the reader's manifold, modulated by their own uncertainty σ^2_{reader}.

The total Lyapunov exponent is:

$$\lambda_{\text{total}} = \lambda_{\text{semantic}} + \lambda_{\text{reader}}, \quad \lambda_{\text{reader}} \propto \sigma^2_{\text{reader}}.$$

Stable interpretations occur when:

$$\lambda_{\text{total}} < \lambda_{\text{crit}}.$$

Example. A reader interpreting "red" as "passion" (literary) vs. "650 nm wavelength" (scientific) introduces divergence, reflected in a higher λ_{reader}.

Physics Intuition. The reader's measurement is like a detector in a chaotic system—its precision determines whether the trajectory remains coherent or scatters into chaos.

Discussion

The framework developed here does not replace existing models of language or machine learning, but repositions them within a geometric and dynamical perspective.

Words are no longer treated as static units, but as attractors within a reconstructed phase space. Sentences become trajectories, and meaning emerges not as a fixed property, but as a path taken through a manifold of possibilities.

This shift has several implications.

First, it provides a natural explanation for instability in language models. Hallucinations are not arbitrary failures, but arise from regions of high curvature or poorly constrained trajectories. This suggests that stability is not achieved through rule enforcement alone, but through shaping the geometry of the underlying space.

Second, it reframes the role of the prompt. Rather than a simple input, the prompt acts as a forcing term—tilting the landscape and selecting among possible trajectories. Small changes in prompt structure can therefore lead to large divergences in output, consistent with the behaviour of nonlinear systems.

Third, it highlights the role of the reader. Meaning is not contained within the output alone, but is reconstructed through interaction with the reader's own manifold. Each interpretation is therefore locally coherent, but globally partial.

Finally, this perspective suggests that language, mathematics, and physical systems may share a deeper structural similarity. All can be understood as processes of reconstruction within finite, constrained spaces, shaped by interaction and stabilisation.

> Words are transducers, turning the tide of stimuli into maps of meaning, their uncertainty the ripples that send the train careening or steadying its course.