

The Attralucian Essays:
Exploring the Finite



First Edition

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Geofinitism: Replacing the Ket with the
Geofinitist Manifold

Kevin R. Haylett

Beyond the Ket Limit

Geofinitism: Replacing the Ket with the Geofinitist Manifold

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Abstract

The quantum state vector $|\psi\rangle$ is the cornerstone of modern physics, yet it rests on a Platonic foundation: the infinite-dimensional Hilbert space. We identify a precise epistemic boundary—**The Ket Limit**—where this formalism transitions from an empirical description to a metaphysical projection. We propose a paradigm shift, rebuilding quantum mechanics on the principles of *Geofinitism*, which asserts that all knowledge is derived from finite measurements converted to symbols. The state is redefined as a finite reconstruction on a thickened statistical manifold, whose resolution is fundamentally bounded by the **Geofinitist Resolution Bound**:

$$\varepsilon_{\text{total}} \geq \max(U_{ex}, 1/b, L/N^{1/d_P}).$$

This framework naturally eliminates quantum singularities, grounds the exponential function in finite combinatorics, and provides a path to a finite quantum gravity. We introduce the **Circular Uncertainty Distribution (CUD)** as the native statistical object of this new foundation, replacing the Gaussian. This is not an interpretation but a reconstruction of quantum mechanics on an epistemically honest, finite basis.

0.1 The Rupture: The Ket Limit

The standard quantum formulation describes a system via a state vector $|\psi\rangle$ in an infinite-dimensional Hilbert space. This assumes infinite resolution, treating constants like π as exact Platonic ideals. However, all empirical access is via finite measurements, which can be formalized as a stream of symbols from a finite alphabet (an *Alphon*), characterized by a spherical boundary and a dynamic resolution.

We define **The Ket Limit** as the epistemic boundary where the informational demand of the ket $|\psi\rangle$ exceeds the capacity of its empirical support:

$$\text{Resolution}(|\psi\rangle) > \varepsilon_{\text{total}} = \max(U_{ex}, 1/b, L/N^{1/d_P})$$

where:

- U_{ex} : Exogenous uncertainty (physical measurement

limit)

- b : Alphabet size (symbolic resolution)
- N : Number of measurement samples
- d_P : Platonic (ideal) dimension of the system
- L : Characteristic scale

Beyond this limit, $|\psi\rangle$ is not a description of reality but a Platonic projection—a map that has mistaken itself for the territory. The phenomena of “wavefunction collapse” and “quantum weirdness” are artifacts of this over-extension.

The New Foundation: Geofinitist Principles

We replace the infinite foundation with the core tenets of Geofinitism:

1. **Primacy of Measurable Numbers:** All physical quantities are measurable numbers M , known only within an uncertainty interval $\pm U$.
2. **The Geofinitist Resolution Bound (GRB):** The precision of any model is fundamentally bounded by the hardest of three constraints: measurement, alphabet, and sampling.
3. **Uncertainty as Dimension:** Uncertainty adds a

“thickness” to geometric objects. The real dimension d_{real} of a system is $d_P + \delta d$, where δd is the dimensional correction from uncertainty.

4. **Statistics as Symbolic Dynamics:** Probability is a useful fiction; the underlying reality is the finite combinatorial geometry of symbolic trajectories.

The State as a Thickened Manifold

We propose that a quantum system is not a wavefunction but a finite, deterministic dynamical system whose full state is hidden. Our access is limited to a finite time-series of Alphon measurements.

The quantum “state” is the attractor manifold reconstructed from this data via a Takens-like embedding. The act of measurement is a coarse-graining that transduces an exogenous “click” into an endogenous data-symbol. The wavefunction Ψ is not a physical wave but an endogenous document that encodes the potentialities inferred from past interactions.

Superposition is reinterpreted not as a physical reality but as a geometric property of this reconstructed manifold—a coordinate representing a state of maximal uncertainty or instability between basins of attraction.

The Combinatorial Heart: The Exponential Demystified

The exponential form, ubiquitous in quantum mechanics (e.g., unitary evolution $e^{-iHt/\hbar}$, Boltzmann factors), is not a fundamental Platonic entity. It emerges inevitably from finite combinatorics.

Derived from counting the arrangements of a finite number N of resources among a finite number M of states under constraint, the most probable distribution takes the form:

$$p_j \approx \frac{1}{Z} e^{-\beta \varepsilon_j} \pm \Delta p_j$$

where the uncertainty Δp_j is constitutive, not a flaw. The infinite-precision exponential is the $\Delta p_j \rightarrow 0$ limit—a *Useful Fiction*, but not the reality. This **Principle of Exponential Emergence** explains its universality: it is the semantic shape of finite multiplicative processes observed through linear symbolic systems.

A Native Object: The Circular Uncertainty Distribution

To embody these principles, we replace the Gaussian with the **Circular Uncertainty Distribution (CUD)**, a probability distribution native to a finite world.

- **Finite Support:** Defined on a circle of circumference $M = \lfloor \pi \cdot b \rfloor$, eliminating the fiction of infinite tails.
- **Inherent Uncertainty:** All parameters $(k_0, \tilde{\sigma}^2)$ are measurable numbers with explicit $\pm U_{ex}$.
- **Resolution-Aware:** Has a probability floor $E_{\min} = 1/(bN)$, acknowledging the smallest meaningfully representable value.

The CUD is not an approximation of the Gaussian; it is a fundamentally different object that reduces to the Gaussian only in the unphysical infinite limit. It is the honest endogenous document for a finite world.

Resolving the Singularities

This framework naturally resolves the pathologies of modern physics:

- **The Hydrogen Atom:** The “stable orbital” is not a planetary orbit or a probability cloud, but a closed geodesic on the reconstructed attractor manifold. Stability is a geometric inevitability, not a quantized law.
- **Quantum Statistics:** The Fermi–Dirac and Bose–Einstein distributions are reformulated with measurable numbers and approximate zeros. The Pauli Exclusion Principle becomes approximate; the Bose–

Einstein condensate is a smooth crossover, not an infinitely sharp phase transition.

- **Quantum Gravity and Singularities:** Space-time is a finite symbolic manifold with the Planck scale as its fundamental alphabet. The concepts of a point of infinite density (black holes, Big Bang) are replaced by finite regions of size $\sim \ell_P$, where density, while immense, is finite. Singularities vanish not by fiat, but as a natural consequence of finite geometry.

Conclusion: The Finite Turn in Physics

We have performed a **Finite Turn** in quantum foundations. By identifying the Ket Limit and rebuilding the theory with Geofinitist principles, we trade a physics of mysterious infinities for a physics of honest finitude. The result is a coherent framework that is epistemically rigorous, mathematically robust, and poised to unify quantum mechanics with gravity on the common ground of finite interaction.

The map, in its finitude and granularity, is the only territory we can ever know. This is the promise of **Finite Quantum Mechanics**.