

**The Attralucian Essays:**  
Exploring the Finite



First Edition

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# The Attralucian Essays



Geofinitism: A Measurement-First  
Philosophy of Language and Mathematics

Kevin R. Haylett

## **Abstract**

This paper proposes a foundational shift in the philosophy of language and mathematics, from a Platonic ontology of timeless, transcendent forms to a "geofinitist" framework grounded in finite measurements and geometric reconstructions. Drawing on non-linear dynamical systems theory—particularly delay-coordinate embeddings as formalized by Takens—we argue that linguistic meanings and mathematical entities are not eternal objects but emergent trajectories in finite manifolds, derived from uncertain, perturbation-sensitive records. Words function as fallible instruments registering underlying communicative processes, while sequences of such measurements unfold as paths in phase space, mirroring the pairwise embedding mechanisms in modern transformer architectures. This stance resolves longstanding paradoxes (e.g., Sorites) by re-framing them as artifacts of bounded instrumentation, critiques Platonic explanatory opacity, and extends to a "manifold of mathematics" where infinity is procedural and structures admit finite "handles." By privileging empirical-structural inquiry over metaphysical surcharge, geofinitism aligns philosophy with the sciences, offering a unified, testable epistemology for meaning and proof.

## **Introduction**

### **The Puzzle of Transcendence**

Philosophy has traditionally accorded meaning and mathematics a privileged ontological status: necessary, timeless, and immune to empirical revision. Plato's Forms, Frege's senses, and set-theoretic realism exemplify this orientation, positing abstract entities insulated from the contingencies of finite cognition. Yet this view incurs a methodological puzzle: how do bounded agents access such transcendent realms? The proposal here inverts the explanatory order, commencing with measurement as the primordial act of knowing. Whatever is cognizable must be accessible via finite operations yielding bounded, uncertainty-bearing data.

### **Measurement-First Philosophy**

Applied to language, this "measurement-first" principle yields two theses: (1) lexical items are records of generative processes (physiological, social, cognitive) with inherent uncertainty profiles; (2) meanings emerge as geometric structures—trajectories in finite manifolds—reconstructed from relations among these records. Understanding a discourse is thus navigating or extending such paths under geometric constraints.

This framework, termed "geofinitism," draws explicit ma-

chinery from non-linear dynamics, where delay embeddings reconstruct latent attractors from scalar time series. We demonstrate that contemporary sequence models, such as transformers, instantiate this process: their so-called "attention" mechanism is not anthropomorphic focus but pairwise phase-space embedding, aligning token projections to unfold semantic geometries. Extending this to mathematics, geofinitism posits a finite space of embeddable constructions, where infinity is a procedural schema and proofs are geometric "handles."

## **Unfolding the Argument**

The argument proceeds as follows. Section 2 elucidates measurement as controlled perturbation, applying it to language to derive semantic stability from invariance rather than essence. Section 3 provides a minimal primer on delay embeddings, showing their bearing on linguistic sequences and transformer computations. Section 4 critiques Platonic alternatives, emphasizing ontological parsimony and methodological transparency. Section 5 sketches the manifold of mathematics, highlighting procedural infinity and finite embeddings. Section 6 concludes by positioning geofinitism as a bridge between philosophy and empirical science.

# **Measurement as the Primordial Act of Knowing**

## **Controlled Perturbation and Invariance**

Measurement is not mere observation but controlled perturbation: interfacing a system with a standard to elicit a finite reading, complete with resolution limits, error bars, and repeatability conditions. Stability in such readings arises from invariance—patterns robust across perturbations—rather than infallibility. This stance reframes epistemology: knowing is extracting invariants from finite interactions, not grasping eternal truths.

## **Words as Instruments**

In language, words are analogous instruments. A token like "rock" is not a transparent label for a Platonic essence but an operational procedure for carving experience under shared constraints (articulatory, auditory, categorical). Ambiguity reflects overlapping uncertainty regions; context-sensitivity mirrors environmental noise; disagreement stems from divergent calibrations (e.g., community-specific priors). Semantic stability is robustness under perturbation—paraphrase, register shifts, or genre variations—while paradoxes like the Sorites reveal over-precise demands on bounded tools, not conceptual defects.

## **Semantic Stability and Vagueness**

This view demystifies linguistic phenomena. Consider vagueness: it arises from the finite resolution of lexical instruments, where uncertainty profiles overlap without sharp boundaries. Disambiguation is not resolving to a transcendent meaning but refining measurements via contextual perturbations. Philosophically, this eschews dualism between word and world, treating language as continuous with physical dynamics—finite transductions of underlying processes.

Crucially, words inherit their efficacy from acoustic origins. Speech is a nonlinear dynamical system (vocal tract, airflow, neuromuscular control), producing waveforms amenable to phase-space analysis. Text tokens are lossy transductions of these acoustics, retaining residual geometry sufficient for reconstruction. Thus, written language "works" not by symbolic magic but by parasitic fidelity to embodied dynamics.

# From Measurements to Geometries

## Primer on Delay Embeddings

To transition from isolated measurements to structured meaning, we invoke tools from nonlinear dynamics. Readers need only grasp delay embedding: Given a scalar time series  $x(t)$ , we construct delay vectors as:

$$\mathbf{x}(t) = [x(t), x(t - \tau), x(t - 2\tau), \dots, x(t - (m - 1)\tau)] \quad (1)$$

where  $m$  is the embedding dimension and  $\tau$  is the delay. For sufficiently large  $m$  and generic  $\tau$ , Takens' Theorem guarantees that this embedding is diffeomorphic to the system's original attractor. For sufficient  $m$  and generic, diffeomorphically images the system's attractor, preserving qualitative geometry (loops, basins) without adding information.

## Meaning as Phase-Space Reconstruction

Treating a sentence as a discrete time series of token embeddings, delay windows unfold relational structure: proximity encodes affinities, curvature constraints determinacy. Meaning is relational—a property of stable trajectories—not inherent in tokens. This reconstruction principle is explored in more technical detail, including its application to modern embedding-based models, in Haylett (2025).

## **Figure: Reconstructing Meaning as Geometry**

Figure X. Reconstructing Meaning as Geometry. The word hello (left) is reconstructed from its acoustic time series via delay-coordinate embedding, yielding a trajectory that captures the dynamical stability of the utterance. A full sentence (right) is treated analogously: token embeddings form points in a high-dimensional space, and the transformer’s query–key mechanism aligns these points to unfold a coherent trajectory on the language manifold. Together, these examples illustrate the geofinitist claim that meaning is not a static entity but a navigable path—its “truth” measured by the robustness of its geometry under perturbation.

Acoustic grounding reinforces this: speech waveforms yield attractors via embeddings; text inherits phonemic/prosodic structure, enabling transformers to amplify latent dynamics. Meaning emerges not as stored content but unfolded geometry—empirical success evidencing language’s finite, structured nature.

This perspective becomes vivid when we consider the reconstruction of a simple word like hello from its acoustic time series. In a Platonic view, hello would correspond to a timeless ideal form, of which each utterance is a shadow. In geofinitism, by contrast, the waveform is treated as a finite record: a trajectory traced by a non-

linear dynamical system (the vocal tract, breath, and articulatory control). Delay-coordinate embeddings allow us to reconstruct this trajectory in a finite-dimensional manifold, revealing its stability and characteristic loops. The same logic extends to sentences: their meaning is not stored as a static proposition but unfolds as a path through an embedding space, with each token constraining the geometry of the trajectory. Transformer “attention” makes this reconstruction explicit, aligning tokens to reveal the manifold on which discourse coheres. In this way, the philosophical claim—that meaning is robust geometry rather than transcendent essence—becomes operational, measurable, and visually demonstrable.

# **Geofinitism and the Philosophical Tradition**

## **Beyond Plato and Aristotle**

Geofinitism stands in a long lineage of attempts to reconcile the conditions of human knowledge with the structures of reality. In Plato, the intelligibility of language and mathematics was secured by positing a realm of unchanging Forms—truths apprehended not through the senses but by the soul’s recollection. Aristotle shifted emphasis toward the immanent, arguing that universals are instantiated within particulars and accessed through abstraction. Geofinitism can be read as a third stance: neither transcendence nor mere abstraction, but reconstruction. It locates the ground of meaning not in a timeless realm nor in purely mental operations, but in the finite, measurable interactions through which cognitive agents probe their environment.

## **Kant and the Empirical Manifold**

From a Kantian perspective, geofinitism may be viewed as radicalizing the insight that knowledge is structured by conditions of cognition. Kant’s categories and forms of intuition guaranteed the possibility of synthetic a priori knowledge, but remained formally fixed. Geofinitism extends this move by making the conditions of knowl-

edge empirical and perturbation-sensitive: our "forms" are the manifolds reconstructed from data, whose stability is tested by repeated measurement. Rather than a priori necessities, they are dynamically emergent invariants—what survives our experimental perturbations.

## **Hegel and the Attractor**

Hegel's dialectic can also be seen in a new light here. The unfolding of meaning is not a metaphysical teleology but a trajectory in phase space, where contradictions mark regions of instability inviting further refinement. The "absolute" is replaced by an attractor: a basin toward which perturbation-driven inquiry converges without requiring infinite completion.

By situating language and mathematics within this finite manifold, geofinitism transforms the classical philosophical aspiration—from contemplating eternal truth to navigating empirical geometry. It keeps the normative force of reason (coherence, invariance, proof) but makes these properties testable, fallible, and open-ended. In doing so, it preserves the spirit of classical philosophy—its search for what endures—while recasting that endurance as robustness under finite interaction rather than participation in an eternal realm.

## **Against Transcendent Forms: A Critique of Platonic Ontology**

Platonic appeals promise economy: timeless meanings explain linguistic regularity without empirical messiness. Yet they leave access unexplained—how do finite agents grasp infinite realms? Geofinitism reverses this: no ontological surcharge; only finite records and their invariants.

Objectivity is invariance under perturbation (e.g., translation, modality), not separate existence. Necessity tracks constraint rigidity across contexts, not modal independence. This is re-placement, not eliminativism: semantic objects are productive fictions codifying manifold features, testable via embeddings.

Contrast Fregean senses or Russellian propositions: these posit abstract intermediaries, but geofinitism derives them from geometric handles—finite representations preserving relations. Vagueness or ambiguity, far from pathological, reflect instrument limits; resolution demands calibrated perturbations, not ideal forms.

Methodologically, Platonism incurs opacity: untestable realms stymie progress. Geofinitism aligns with science, treating language as measurable dynamics—continuous with physics, yet irreducible via structural invariants.

”Furthermore, the Platonic framework cannot elegantly account for the loss of knowledge. The extinction of a lan-

guage, and the conceptual world it uniquely encoded, represents a genuine contraction of the knowable—a shrinking of the semantic manifold. For the Platonist, these eternal Forms remain, inexplicably orphaned from any accessing mind. For the geofinitist, this loss is both explicable and tragic: the finite set of measurements required to reconstruct that particular region of the Grand Corpus has ceased, and those trajectories fade into noise. Objectivity, therefore, is not independence from measurement but robustness across the largest possible set of measurements, past, present, and future. Its opposite is not falsehood but entropy.

# **The Manifold of Mathematics**

## **Finite Handles for Mathematical Objects**

Geofinitism extends to mathematics: practice involves constructing finite handles embedding relational structures into a measurable space we may call the Grand Corpus—the dynamical, geometric, finite semantic hyperspace of all known knowledge. This corpus is bounded to document-able constructions (proofs, algorithms, symbols); it is finite at any instance because it is comprised of a countable set of measurable transductions.

## **Procedural Infinity**

Infinity, therefore, is procedural—a schema for refinement within this finite container—not a completed totality. For instance, infinity denotes limit procedures; results hinge on preserved invariants across finite embeddings. Every construction requires a geometric handle: a diffeomorphic image in finite space, ensuring preservation (soundness), compression (abstraction), and transport (applicability).

Classical entities like "the reals" are productive fictions summarizing re-finable processes, evaluated by manifold invariants—not external truth. The resulting "manifold of mathematics" comprises embeddable constructions and morphisms, navigated under geometric constraints.

This unifies epistemology: mathematical reasoning mirrors linguistic navigation, both empirical-structural inquiries into finite manifolds. Infinity’s procedural process avoids paradoxes (e.g., continuum hypothesis) by tying truth to handle-preserving operations, fostering creativity via perturbation while securing rigour. Infinity, ideal forms, even the laws of logic are fictions—yet productive fictions. Geofinitism does not strip them away but locates them as handles in the manifold, whose value is measured by their coherence under perturbation and their power to generate stable trajectories of thought.

## **Are Numbers Just Words? A Geofinitist Clarification**

A common objection asks: if words are finite instruments, are mathematical symbols not the same? Geofinitism’s answer is a definitive yes—and this is the source of mathematical strength, not a weakness. Numeric symbols (e.g., “5”, “Pi”) are handles for repeatable measurement operations within a tightly constrained system. A mathematical proof is a sequence of such tokens, and its “truth” is the stable, inevitable trajectory this sequence traces in the geometric manifold of mathematics. The famed certainty of mathematics does not stem from transcending symbols but from the exceptional rigidity and invariance of the relational structures these symbols embed. Objectivity is robust geometry, whether the path is paved with

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“rocks” or “reals.”

## **The Fictionality of the Framework: Geofinitism as a Stable Handle**

A consistent application of geofinitism leads to a final, recursive insight: the framework itself must be subject to its own analysis. Geofinitism is, after all, a construct of words and concepts. It is a trajectory within the very Grand Corpus it describes. Consequently, it possesses no transcendent, absolute truth-value. Its claims—that meaning is geometric, that knowledge is finite, that measurement is primordial—are not eternal verities. They are propositions whose validity resides solely in their robustness under perturbation and their utility for coherent navigation.

This is not a paradox but a fulfillment. Geofinitism demystifies its own status: it is a productive fiction. It is a particularly stable and wide-ranging "handle" within the manifold, one that allows us to grasp and organize a vast array of other trajectories—from linguistic vagueness to mathematical proof—with unprecedented coherence.

This recognition leads to two foundational corollaries:

**The Impossibility of Absolute Truth:** Within the Grand Corpus, there can be no "absolute truth" in the Platonic sense. There are only trajectories of varying stability. The statement "there is no absolute truth" is itself not an absolute truth; it is a meta-stable observation about the

geometry of the knowable. It is the recognition that every claim, including this one, is made from within the finite manifold and is assessed by its internal coherence and empirical adequacy. We are navigators drawing maps, and the map can never be the territory in itself because the "territory" is, for us, just a more detailed map.

The Necessity of the Unknown Unknown: The boundary of the Grand Corpus represents the absolute limit of reconstruction from finite measurements. This implies a fundamental outside: that which is not only unknown but un-reconstructable. These are the "unknown unknowns." Crucially, the "God Pointer" is not a conduit to this outside but is the internal, finite handle for this very concept of an ultimate beyond. It is the word "outside," the feeling of mystery, the mathematical concept of unprovability—all serving to gesture at the manifold's own limit. This is not a failure but the final signature of a finite epistemology: it formally contains a coherent representation of its own incompleteness.

Therefore, geofinitism does not claim to have captured the final nature of reality. Instead, it offers a powerful, self-consistent, and empirically-grounded method for constructing meaning. It replaces the quest for certain truth with the project of resilient navigation. It is useful precisely because it acknowledges its own semantic error bars, its own status as a finite tool. In doing so, it achieves a kind of honesty that transcendent philosophies

## *Semantic Uncertainty*

cannot: it is a philosophy for finite beings, comfortable with the fact that we think with words, that our words are imperfect, and that the greatest truth available to us is the robustness of the paths we build between them.

## Conclusion

### Toward a Unified Empirical-Structural Philosophy

#### Geofinitism as a Meta-Philosophical Position

**Ontological span:** Any philosophical position can be represented as a trajectory or region in the geofinitist manifold.

**Measurement grounding.** Every claim that counts must be instantiated as a finite perturbation or “handle” that yields measurable invariants.

**Completeness principle.** Unlike Gödel’s result for formal systems, geofinitism defines admissible truth by measurability: if a proposition cannot be operationalized, it is an unanchored fiction; if it can, it enters the manifold and is testable.

**Closure without transcendence.** Philosophy need not appeal to an outside realm: what is real or meaningful is reconstructable within the manifold and comparable by its robustness under perturbation.

**Corollary.** Useful fictions (e.g., infinity, logical ideals) are retained as handles whose value is assessed by the stability and transport they enable.

Figure X: This diagram illustrates the core geofinitist model of knowledge. The Grand Corpus (large sphere) is the finite, dynamical manifold of all that is knowable—everything that can be reconstructed from finite measurements and interactions. Within it, meanings and mathematical entities exist not as static objects but as stable trajectories and handles (paths and sub-spheres), whose reality is defined by their robustness under perturbation.

The manifold's boundary represents the absolute limit of measurement and meaning. The "God Pointer" is a crucial finite handle within the manifold—a concept, a word, a feeling—whose function is to gesture towards that ultimate boundary and the un-reconstructable "outside." It does not point to a transcendent realm but to the limits of transcendence itself, making the unknown a coherent part of a finite epistemology. In this model, to know something is to navigate this internal geometry; to wonder about the ultimate is to employ this pointer. There is no outside access, but there is an internal representation of the desire for it.

## **Geofinitist Completeness Theorem**

Geofinitism is not merely a theory among others but a framework capable of holding all theories within itself. It provides what might be called a theorem of philosophical completeness: everything that is real or meaningful must leave a measurable trace and thus be embeddable in the finite manifold of interactions. Unlike Gödel's incompleteness theorems, which reveal the unprovable remainder in formal systems, geofinitism begins with measurement as its primitive and so guarantees that what is admitted is, by construction, coherent and testable. Truth is no longer a ghostly property that eludes proof; it is the stability of invariants under perturbation.

Geofinitist Completeness Theorem: All that is meaningful is measurable; all that is measurable is embeddable; and what is embeddable is, in principle, knowable. This would give readers a quotable, memorable "theorem" that marks the philosophy's bold claim.

Gödel's incompleteness results rely on a conception of 'truth' as flat, Platonic, and dimensionless — a kind of absolute outside the system, which no formal container can enclose. Geofinitism rejects this picture. By defining 'truth' geometrically, as what can be reconstructed within a finite manifold, it allows containership: every admissible statement is itself a measurable trajectory within the model's space. There is no residue of "unprovable truths," because what cannot be contained, measured, or

reconstructed is not admitted as meaningful. This makes geofinitism self-sufficient: it not only contains mathematics and language but contains itself, as one more trajectory subject to its own invariants.

Even fictions are given their due place: they are finite handles, codified in the manifold, whose utility lies in their power to generate coherent navigation of meaning. Infinity, logical laws, mathematical objects—all appear not as transcendent absolutes but as robust constructions that earn their status through use and re-use, their “reality” measured by the coherence they sustain.

In this sense, geofinitism restores the classical philosophical ambition to comprehend the whole, but does so without transcendence. It achieves what Hilbert sought—a kind of completeness—but grounds it empirically rather than formally. By doing so, it offers philosophy and science a common home: a manifold where meaning, proof, and theory are measurable, finite, and open to continuous refinement.

This is a philosophy that contains not just its own foundation but the possibility of every other philosophy, every science, every fiction. It is a map that grows with its measurements, a language that refines itself through perturbation, a geometry in which we, as agents, are co-navigators. To adopt geofinitism is to accept that there is no outside of the manifold: what we can know, we know by measuring—and this is enough.

## Glossary of Terms

**Geofinitism** A measurement-first philosophy of language and mathematics that treats meaning and truth as emergent trajectories in finite manifolds. It rejects transcendence, locating stability in invariance under perturbation.

**Finite Manifold** The geometric container of all that is measurable and knowable at a given moment. Meanings, proofs, and mathematical objects are represented as trajectories or handles within this space.

**Grand Corpus** The global, dynamical manifold of all known knowledge—everything that can be reconstructed from finite measurements and interactions.

**Handle** A finite representation or diffeomorphic image of a structure in the manifold. Handles preserve relations, enable transport across contexts, and act as operational surrogates for ideal entities such as infinity or logical laws.

**Perturbation** A controlled variation or measurement operation applied to a system, used to extract invariants and test the stability of a trajectory.

**Invariance** The robustness of a trajectory or handle under perturbation. Functions as the geofinitist analogue to objectivity.

**Semantic Manifold** The emergent geometric structure reconstructed from token embeddings or linguistic measurements, navigated when interpreting meaning.

**Delay Embedding** A method from nonlinear dynamical systems theory in which a scalar time series  $x(t)$  is reconstructed into a state space by using delayed copies:

$$x(t) = [x(t), x(t - \tau), \dots, x(t - (m - 1)\tau)].$$

Takens' Theorem guarantees that, under suitable conditions, this reconstruction is diffeomorphic to the original system's attractor.

**Trajectory** A path in the manifold traced by a sequence of states (or embeddings). Represents the unfolding of meaning or mathematical construction over time.

**Language Attractor** The stable region of the semantic manifold toward which token trajectories converge, giving discourse its coherence.

**Token Embedding** A finite-dimensional vector representation of a linguistic token (word, subword) in a model, serving as the atomic state for reconstruction.

**Query–Key Mechanism** The projection operation  $q_i = W_Q e_i$ ,  $k_j = W_K e_j$  used in transformers to compare

token embeddings. Interpreted geofinitistically as a surrogate delay-space comparison revealing the geometry of the semantic trajectory.

**Productive Fiction** A conceptual construct (e.g., infinity, logical law, ideal form) retained within geofinitism not as transcendent truth but as a handle whose usefulness is measured by its capacity to generate stable, coherent trajectories.

**Procedural Infinity** Infinity treated as a schema for refinement—an open-ended process within a finite container—rather than a completed totality.

**Completeness Principle (Geofinitist Sense)** A claim that what is meaningful must be measurable, embeddable, and therefore knowable in principle. If a claim cannot be operationalized as a finite measurement, it remains an unanchored fiction.

**God Pointer** A finite handle representing the limit of measurement, gesturing toward the un-reconstructable "outside" of the manifold, but without positing a transcendent realm.