FSM Information Theory: Symbolic Containment and Functional Trajectories

A Working Trajectory

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Abstract

This chapter develops a preliminary formal comparison between Shannon information and a Geofinite treatment of information as finite symbolic containership. Shannon's mathematical theory of communication treats information as uncertainty over selectable messages or symbols. This produces a powerful and historically successful abstraction in which the symbol is treated as a dimensionless alternative. The Geofinite position explored here begins from a different premise: a symbol is a finite geometric containment, instantiated through measurement at a generonic boundary. Under this treatment, a symbol does not merely distinguish; it contains an uncertainty geometry and carries an unrecoverable generative path.

The chapter first reviews the historical setting of telegraphy, Hartley information, Shannon entropy, binary digits, and the later inheritance of information-theoretic language in computation and quantum theory. It then formalises Shannon's framework, including entropy, choice of logarithmic base, mutual information, channel capacity, and the usual base invariance of dimensionless symbols. A corresponding Geofinite formalism is then introduced, in which finite symbols possess containment geometry, the Alphon is represented as a minimum isotropic uncertainty sphere, and base is no longer merely a unit choice but a structural property of symbolic measurement.

The Equivalent Alphonic Pi construction is used as a worked result. In the implemented frame, the Alphonic Circle Condition gives a minimum stable symbolic base

$$B_{\min} = \lceil 10/\pi \rceil = 4,$$

suggesting that planar circular closure requires more than binary distinction. The result is interpreted cautiously as evidence that not all measurable differences are geometric, and not all geometric properties are reducible to distinction. The discussion closes with future directions for a Geofinite information theory, including spherical containership, base-dependent geometry, path-loss entropy, language as symbolic containment, and possible reinterpretations of Hilbert-space probability as a projected management of uncertainty rather than a primary geometry.

Status of this Document

This is a working research draft. It does not claim to replace Shannon information theory, quantum mechanics, or standard mathematical foundations. Its purpose is to make explicit what is removed when symbols are treated as dimensionless alternatives, and to develop a candidate formalism in which finite symbols are treated as geometric containment objects. The chapter is written as a scaffold for later refinement, mathematical correction, and experimental development.

Contents

Chapter 1

Geofinite Information and Symbolic Containership

1.1 Historical Introduction

The twentieth-century mathematical treatment of information emerged from concrete engineering problems. Telegraphy, telephony, modulation, noise, bandwidth, coding, and switching all required methods for representing and transmitting signals with increasing efficiency. In this historical setting, information was not first introduced as a general metaphysics of meaning. It was introduced as an engineering quantity.

Nyquist's work on telegraph speed examined constraints on signal transmission and the maximum rate at which intelligence could be sent through a telegraph system [1]. Hartley's 1928 paper, *Transmission of Information*, developed a quantitative measure of information based on physical rather than psychological considerations [2]. Hartley's key idea was that information could be associated with the number of possible symbol selections. If a system permits a selection from N equiprobable alternatives, the amount of information may be measured logarithmically:

$$H_0 = \log_b N. \tag{1.1}$$

This is sometimes called Hartley information. It anticipates the central role of logarithms in Shannon's later theory.

Shannon's 1948 paper, *A Mathematical Theory of Communication*, placed these ideas into a general probabilistic framework [3]. Shannon explicitly separated the engineering problem of communication from semantic interpretation, writing that semantic aspects are "irrelevant to the engineering problem." The significant fact, for Shannon's formalism, was that a message is selected from a set of possible messages. The receiver and channel must be able to handle all possible selections, not merely the particular selection that occurs.

This move was extraordinarily powerful. It enabled a general theory of communication in which messages could be encoded, transmitted, corrupted by noise, and decoded. It also enabled the concept of an optimal code length and the mathematical analysis of redundancy and compression. However, this same move also introduced a constraint: the symbol was treated as a selectable alternative, not as a finite geometric object.

The binary digit, or bit, was introduced by Shannon as a unit of information when logarithms are taken base 2 [3]. The historical success of binary representation was later reinforced by the physical convenience of two-state digital electronics. A transistor switch, relay, flip-flop, or memory cell can be idealised as supporting two distinguishable states. In this context, base 2 became not merely a mathematical convenience but the de facto measurement language of computation.

A related development occurred in quantum theory. Born introduced a statistical interpretation of the wavefunction in 1926, associating squared amplitudes with probabilities [4].

Von Neumann later formalised quantum mechanics using Hilbert spaces, operators, projection, and density matrices [5, 6]. In modern quantum information theory, qubits are represented by unit vectors in complex Hilbert space, and measurement probabilities are obtained from inner products, projectors, or traces [7]. This is a highly successful mathematical framework. Yet it also relocates measurement into a projected probability space.

The Geofinite position developed in this chapter does not deny the effectiveness of Shannon information, digital computation, or Hilbert-space quantum mechanics. Instead, it asks what was excluded at the beginning. If a symbol is not dimensionless, but finite and geometric, then base, information, entropy, probability, and measurement must be reconsidered. The central question becomes:

> What finite geometry must a symbol contain before it can be treated as a selectable alternative, encoded message, bit, or measurement outcome?

This question motivates the formal comparison that follows.

1.2 Notation and Working Commitments

The following notation is used throughout this chapter.

Symbol	Meaning
$\mathcal{A}_B$	Alphabet or symbol set of base B, with $\lvert\mathcal{A}_B\rvert = B$.
σ_i	The i-th symbol in an alphabet.
X	A random variable over a symbol or message set.
p_i	Probability of symbol or message x_i.
$H_b(X)$	Shannon entropy measured using logarithmic base b.
Γ	Generonic boundary map from pre-symbolic interaction/path to finite symbol.
Ω	Space of possible generative paths or pre-symbolic interactions.
C_i	Finite containment geometry associated with symbol σ_i.
α	Alphonic Limit, the smallest admissible finite measurement resolution.
S_α	Minimum isotropic uncertainty sphere at scale α.
Q_α^n	Orthogonal grid cell or cube in n dimensions at scale α.
B	Symbolic base or number of distinguishable symbol positions.

The Geofinite framework used here adopts the following working commitments.

(G1) Measurement produces finite symbols.

(G2) A finite symbol is not a dimensionless label but a containment of uncertainty.

(G3) The smallest admissible symbol is not a point but a finite uncertainty region.

(G4) At the Alphonic Limit, absent further directional distinction, the least-assumptive containment geometry is isotropic and is therefore represented by a sphere.

(G5) A base is not merely an encoding convention when symbols have geometry. It changes the available symbolic containment structure.

(G6) Entropy may be reinterpreted, at a deeper level, as non-recoverable generative path-loss after finite symbol instantiation.

These commitments are not presented as completed axioms. They are working principles for the formal construction.

1.3 Shannon's Mathematical Formalism

1.3.1 Messages, Symbols, and Probability

Let $\mathcal{A} = \{a_1, a_2, \ldots, a_N\}$ be a finite alphabet or message set. In Shannon's treatment, a source emits symbols or messages according to a probability distribution

$$\mathbb{P}(X = a_i) = p_i, \qquad p_i \geq 0, \qquad \sum_{i=1}^{N} p_i = 1. \tag{1.2}$$

The symbol a_i is treated as a distinguishable alternative. Its internal geometry is not part of the formalism.

The entropy of the source is

$$H_b(X) = -\sum_{i=1}^{N} p_i \log_b p_i. \tag{1.3}$$

The logarithmic base b fixes the unit. For $b = 2$, the unit is the bit. For $b = e$, the unit is the nat. For $b = 10$, the unit is the Hartley or decimal digit in some contexts.

When all N messages are equiprobable, $p_i = 1/N$, equation (1.3) reduces to

$$H_b(X) = \log_b N, \tag{1.4}$$

which recovers Hartley's measure.

1.3.2 Base as Unit Choice

In Shannon information theory, changing the logarithmic base changes only the unit of measurement:

$$\log_c p = \frac{\log_b p}{\log_b c}. \tag{1.5}$$

Therefore,

$$H_c(X) = \frac{H_b(X)}{\log_b c}. \tag{1.6}$$

This is the standard base-invariance of Shannon information: the underlying uncertainty is considered the same, while its numerical expression changes with the chosen unit.

This base-invariance depends on the symbol being treated as dimensionless. The symbol has identity as a selectable alternative, but not volume, curvature, orientation, or containment geometry.

1.3.3 Channels and Mutual Information

A communication channel may be represented by conditional probabilities

$$p(y_j \mid x_i), \tag{1.7}$$

where x_i is an input symbol and y_j is an output symbol. The joint distribution is

$$p(x_i, y_j) = p(x_i)p(y_j \mid x_i). \tag{1.8}$$

The conditional entropy is

$$H(Y \mid X) = -\sum_i \sum_j p(x_i, y_j) \log_b p(y_j \mid x_i), \tag{1.9}$$

and the mutual information is

$$I(X;Y) = H(Y) - H(Y \mid X) = H(X) - H(X \mid Y). \tag{1.10}$$

The channel capacity is

$$C = \max_{p(x)} I(X;Y), \tag{1.11}$$

where the maximisation is taken over possible input distributions.

This formalism measures how much uncertainty about one selectable symbol is reduced by observing another selectable symbol. It does not ask whether the symbols themselves possess finite geometry.

1.3.4 Compression

In Shannon's framework, compression arises from redundancy. A source with non-uniform or correlated symbol probabilities can be encoded more efficiently than a naive fixed-length code would suggest. For long sequences produced by a stationary source, the optimal average code length approaches the entropy rate of the source.

If $X_1, X_2, \ldots, X_n$ is a sequence from a source, the entropy rate is often written

$$\bar{H}(X) = \lim_{n\to\infty} \frac{1}{n} H(X_1, X_2, \ldots, X_n), \tag{1.12}$$

when the limit exists.

Compression then concerns the reduction of symbolic length while preserving recoverability of the intended symbol sequence under a specified code. It is not, in the Shannon formalism, a theory of recovering the generative path by which the symbol sequence came into being.

1.3.5 The Shannon Projection

For later comparison, Shannon's treatment may be written abstractly as a projection

$$\Pi_S : \quad \text{symbolic process} \longmapsto (\mathcal{A}, p), \tag{1.13}$$

where $\mathcal{A}$ is the set of distinguishable alternatives and p is a probability distribution over them.

The internal geometry of $\sigma_i \in \mathcal{A}$ is not part of $(\mathcal{A}, p)$. The symbol is flattened into a countable alternative.

1.4 Hilbert-Space Probability as a Related Projection

Quantum mechanics is not Shannon theory. However, the modern language of quantum information often places quantum states, probabilities, measurement outcomes, and bits or qubits into a shared mathematical environment.

A pure quantum state is represented by a unit vector $|\psi\rangle$ in a complex Hilbert space $\mathcal{H}$:

$$\langle \psi \mid \psi \rangle = 1. \tag{1.14}$$

An observable A is represented by a self-adjoint operator. In the spectral form,

$$A = \sum_k a_k P_k, \tag{1.15}$$

where P_k are projection operators. The probability of obtaining outcome a_k is given by the Born rule:

$$\mathbb{P}(a_k \mid \psi) = \langle \psi \mid P_k \mid \psi \rangle. \tag{1.16}$$

For a density matrix ρ, the probability becomes

$$\mathbb{P}(a_k \mid \rho) = \mathrm{Tr}(\rho P_k). \tag{1.17}$$

The von Neumann entropy is

$$S(\rho) = -\mathrm{Tr}(\rho \log \rho), \tag{1.18}$$

with the logarithmic base again determining the unit.

From the Geofinite perspective developed here, this formalism may be read as a projected probability geometry. It represents uncertainty over measurement outcomes in a mathematically powerful state space. However, it does not begin by assigning finite physical or symbolic volume to the measurement symbol itself. The state space is not the same object as the finite geometric containment of the measurement at the Alphonic Limit.

This distinction is important. The claim is not that Hilbert space is incorrect. The claim is that it may be a projection of measurement uncertainty after finite geometric containership has already been abstracted away.

1.5 The Geofinite Formalism

1.5.1 Finite Symbolic Containment

The Geofinite treatment begins by refusing to treat the symbol as dimensionless.

Definition 1.1 (Finite Symbol)**.** *A finite symbol is an ordered pair*

$$\Sigma_i = (\sigma_i, C_i), \tag{1.19}$$

where σ_i is a distinguishable mark and $C_i \subset \mathbb{R}^3$ is its finite containment geometry.

In Shannon's formalism, only σ_i is retained. In the Geofinite formalism, C_i is not optional. It is part of what the symbol is.

Definition 1.2 (Geofinite Alphabet)**.** *A Geofinite alphabet of base B is a finite set*

$$\mathcal{A}_B^G = \{(\sigma_0, C_0), (\sigma_1, C_1), \ldots, (\sigma_{B-1}, C_{B-1})\}, \tag{1.20}$$

where each C_i is a finite containment region and $|\mathcal{A}_B^G| = B$.

The same visible glyph can participate in different Geofinite alphabets because its containment role depends on the base and measurement frame. Thus the mark 0 is not intrinsically numerical, empty, binary, decimal, or null. It becomes one of these through its admissible containment role inside a symbolic system.

1.5.2 The Generonic Boundary

Let Ω denote a space of possible pre-symbolic interactions or generative paths. The Generonic Boundary is represented as a map

$$\Gamma_{\alpha,B} : \Omega \longrightarrow \mathcal{A}_B^{G*}, \tag{1.21}$$

where $\mathcal{A}_B^{G*}$ is the set of finite strings over the Geofinite alphabet and α is the Alphonic Limit.

The important property is many-to-one containment:

$$\Gamma_{\alpha,B}^{-1}(s) = \{\omega \in \Omega : \Gamma_{\alpha,B}(\omega) = s\}. \tag{1.22}$$

For a finite symbol string s, the fibre $\Gamma_{\alpha,B}^{-1}(s)$ contains all pre-symbolic paths that instantiate as s. Unless the fibre is a singleton, the originating path is not recoverable from the symbol.

Definition 1.3 (Generonic Path Loss). *Given a generonic map $\Gamma_{\alpha,B} : \Omega \to S$, the path-loss associated with a symbol $s \in S$ is the non-recoverable fibre*

$$L_\Gamma(s) = \Gamma^{-1}_{\alpha,B}(s). \tag{1.23}$$

If a probability measure exists on Ω, the expected path-loss may be represented by the conditional entropy

$$H(\Omega \mid S) = \sum_{s \in S} p(s) H(\Omega \mid S = s). \tag{1.24}$$

This provides a formal way to express the earlier intuition: entropy may be interpreted more deeply as non-recoverable path-loss after finite symbol instantiation. Shannon entropy measures uncertainty over instantiated symbols. Generonic path-loss measures what cannot be recovered about the originating paths after instantiation.

1.5.3 The Alphon as Minimum Symbolic Sphere

At the Alphonic Limit, no further directional distinction is available. Therefore the least-assumptive containment geometry is isotropic.

Definition 1.4 (Alphon). *The Alphon at resolution $\alpha > 0$ is the minimum isotropic uncertainty container*

$$S_\alpha(x) = \{y \in \mathbb{R}^3 : \|y - x\|_2 \le \alpha/2\}. \tag{1.25}$$

It represents the smallest admissible spherical containment of a finite symbol at the measurement limit.

The Alphon is not a point. It is a finite uncertainty sphere. In this chapter, the sphere is treated as prime in the limited sense that it is the least directionally biased containment at the Alphonic Limit.

1.5.4 Grid, Cube, and Sphere

Measurement also requires addressability. The simplest orthogonal address container in n dimensions is the grid cell

$$Q^n_\alpha(x) = \prod_{k=1}^{n} \left[x_k - \frac{\alpha}{2}, x_k + \frac{\alpha}{2} \right]. \tag{1.26}$$

In two dimensions this is a square; in three dimensions it is a cube.

The diagonal of the square is

$$d_2 = \alpha\sqrt{2}, \tag{1.27}$$

and the body diagonal of the cube is

$$d_3 = \alpha\sqrt{3}. \tag{1.28}$$

These values are not merely irrational numbers. In the Geofinite reading they express a mismatch between orthogonal symbolic address and isotropic uncertainty containment. A unitary edge and a diagonal path are not the same geometric containment, even if a conventional symbolic system attempts to collapse both into scalar length.

This is the root-two/root-three problem in the present framing: a finite minimum measurable distance creates both a cube-like address frame and a sphere-like uncertainty frame. Their relations cannot be made identical without projection or loss.

1.5.5 Geofinite Information Object

A complete Geofinite information object may be represented as

$$\mathcal{I}_G = (\mathcal{A}_B^G, p, C, \Gamma_{\alpha,B}, \alpha), \tag{1.29}$$

where:

- $\mathcal{A}_B^G$ is the finite geometric alphabet,
- p is a probability distribution when one is available,
- C assigns containment geometry to symbols,
- $\Gamma_{\alpha,B}$ maps pre-symbolic paths to finite symbols,
- α is the Alphonic Limit.

Shannon information is then recovered as a projection:

$$\Pi_S(\mathcal{I}_G) = (\mathcal{A}_B, p). \tag{1.30}$$

The projection Π_S discards containment geometry, the generonic map, and the Alphonic Limit. This is precisely why Shannon's formalism is mathematically powerful and precisely why it cannot recover finite symbolic geometry without additional structure.

1.6 Base Invariance and Base Dependence

1.6.1 Classical Base Invariance

Let x be an abstract number. A representation function in base B may be written as

$$R_B(x) = s_B, \tag{1.31}$$

where s_B is a finite or infinite symbol string in alphabet $\mathcal{A}_B$. A value function

$$V_B(s_B) = x \tag{1.32}$$

recovers the abstract value. If $s_B = R_B(x)$ and $s_C = R_C(x)$, then

$$V_B(s_B) = V_C(s_C) = x. \tag{1.33}$$

This is ordinary numerical base invariance.

It depends on the symbols being treated as labels. The string changes, but the abstract value is preserved.

1.6.2 Geofinite Base Dependence

In a Geofinite alphabet, however, each symbol has containment geometry. A base transformation does not merely change labels. It changes the available containment partition:

$$\mathcal{A}_B^G \not\cong \mathcal{A}_C^G \qquad \text{in general.} \tag{1.34}$$

Even if two strings represent the same abstract value under conventional value functions, their containment geometries differ:

$$C_B(s_B) \neq C_C(s_C) \qquad \text{in general.} \tag{1.35}$$

Thus the formal claim becomes:

Principle 1.1 (Split Base Invariance)**.** *Abstract numerical value may be base-invariant under conventional conversion, but finite symbolic geometry is not generally base-invariant.*

This principle is one of the central results of the present chapter.

1.7 Equivalent Alphonic Pi

1.7.1 Classical Pi Across Bases

Classically, π can be expressed in any base. For example:

$$\pi_{10} = 3.1415926535\ldots, \tag{1.36}$$

$$\pi_{16} = 3.243f6a8885\ldots, \tag{1.37}$$

$$\pi_{2} = 11.001001000011111\ldots. \tag{1.38}$$

These are different symbolic representations of the same abstract ratio of circumference to diameter.

Under ordinary base conversion, this is unproblematic. The glyphs change, but the abstract value remains invariant.

1.7.2 Equivalent Alphonic Pi Definition

The Equivalent Alphonic Pi construction changes the question. It asks what circular geometry looks like when expressed relative to an alphonic unitary of base B while retaining the conventional decimal reference unit as a comparison.

Define

$$\mathrm{EAP}(B) = \pi \frac{B}{10}. \tag{1.39}$$

Here B is the symbolic base, and 10 is the inherited decimal reference unit. The factor $B/10$ is not a claim that decimal is fundamental. It makes explicit the decimal tether that ordinarily remains hidden.

1.7.3 Alphonic Circle Condition

The Alphonic Circle Condition is written

$$\mathrm{EAP}(B) \geq 1. \tag{1.40}$$

Substituting equation (1.39) gives

$$\pi \frac{B}{10} \geq 1. \tag{1.41}$$

Therefore

$$B \geq \frac{10}{\pi}. \tag{1.42}$$

Since B must be an integer base,

$$B_{\min} = \left\lceil \frac{10}{\pi} \right\rceil = 4. \tag{1.43}$$

Observation 1.1 (Base-4 Threshold)**.** *In the EAP construction, base 4 is the minimum stable symbolic base satisfying the Alphonic Circle Condition.*

This is the key computational and formal observation.

1.7.4 Interpretation of the Threshold

The base-4 result should not be interpreted as saying that a circle has four sides. Instead, it suggests that a circle cannot be measurably contained as a closed planar geometry until the symbolic ruler has at least four units of distinction at the alphonic resolution.

Binary distinction can distinguish opposition:

$$0 \mid 1. \tag{1.44}$$

But opposition is not closure. Base three provides a further mediating symbol, but still does not satisfy equation (1.40). Base four is the first frame in which departure, orientation, opposition, and return can be minimally separated.

A useful heuristic hierarchy is therefore:

$$B = 2 : \text{distinction/opposition,} \tag{1.45}$$
$$B = 3 : \text{mediation/relational discrimination,} \tag{1.46}$$
$$B = 4 : \text{first planar geometric closure,} \tag{1.47}$$
$$B \geq 4 : \text{stable symbolic circular containment.} \tag{1.48}$$

This hierarchy remains a conjectural interpretation, but it gives formal language to the observed threshold.

1.7.5 Observed EAP Values

The following table records the leading-symbol behaviour from the EAP generator discussed in the research session. The interpretation column is framed in terms of whether the base satisfies the closure condition.

Base	Pi Lead	EAP Lead	EAP Position	Status
4	3	1	33.3%	Stable
5	3	1	25.0%	Stable
6	3	1	20.0%	Stable
7	3	2	33.3%	Stable
8	3	2	28.6%	Stable
9	3	2	25.0%	Stable
10	3	3	33.3%	Stable
11	3	3	30.0%	Stable
12	3	3	27.3%	Stable
16	3	5	33.3%	Stable
20	3	6	31.6%	Stable
24	3	7	30.4%	Stable
28	3	8	29.6%	Stable
32	3	a	32.3%	Stable
36	3	b	31.4%	Stable

For base 36, the generator produced the classical base-36 representation

```
pi ~= 3.53i5ab8p5fsa5jhk72i8asc47wwzlacljj9zn98ltxm61vyms1
```

and the Equivalent Alphonic Pi value

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EAP ~= b.b5exfivbcdmm5jr7wnf8a17me3arbu25jxed4iuze5tel72a05
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The difference between these strings is central. The first is classical π represented in base 36. The second is π rescaled into a base-36 alphonic unitary using equation (1.39). The geometry is not destroyed, but the symbolic containment frame changes.

1.8 Containership, Enclosure, and the Sphere

1.8.1 Containment Before Distinction

The morning insight motivating this chapter may be stated as follows:

Principle 1.2 (Containment Priority)**.** *At the Alphonic Limit, measurement is not first the refinement of precision. It is the finite containment of a relation such that the relation can be distinguished at all.*

This principle reverses the usual order. In Shannon's framework, distinction comes first: a message is selected from a set. In the Geofinite framework, containment comes first: a symbol contains the uncertainty geometry of a finite measurement.

Distinction is then a property of contained relations, not the primitive foundation.

1.8.2 The Sphere as Prime Limit

The sphere is prime in this framework because it is the minimum isotropic containment at the limit of directional indistinguishability. This does not mean that all geometry is literally spherical at all scales. It means that when no further directional distinction is available, the least biased uncertainty container is spherical.

A square, cube, grid, vector basis, or Hilbert-space representation may be extremely useful. But each is already structured. Each introduces axes, dimensions, coordinates, or basis states.

The sphere is therefore used as the first containment, while the cube or grid is used as the first addressable containment.

1.8.3 Binary Flattening

Base 2 is sufficient for distinction but insufficient for geometric enclosure in the EAP construction. It may flatten the measurement into two opposed symbolic states:

$$\{0, 1\}. \tag{1.49}$$

If interpreted geometrically, these two states can represent opposition, a line segment, or two sides of a collapsed projection. They do not provide enough orientation to contain circular return.

This leads to a second principle:

Principle 1.3 (Binary Compression)**.** *Binary distinction is not geometric containment. It is the compression of containment into opposition.*

This does not make binary representation false. It makes it a projection.

1.9 Language as Symbolic Containment

The present formalism also applies to language. A word is not merely a transmitted label. It is a symbolic container in which meaning may stabilise for a reader.

Let W be a set of words and $\mathcal{M}$ a semantic phase space. A word $w \in W$ may be associated with a semantic containment basin

$$B_w \subset \mathcal{M}. \tag{1.50}$$

A reader r does not receive B_w directly. The reader reconstructs a basin through their own history, context, language, and interpretive frame:

$$M_r(w) = R_r(B_w), \tag{1.51}$$

where R_r is the reader's reconstruction map.

This gives a formal expression to the phrase "the reader is the author." The sentence does not transfer complete meaning. It provides a containment structure that the reader re-instantiates.

Definition 1.5 (Slow Noun). *A slow noun is a symbol w_t whose semantic containment basin stabilises only through repeated use, refinement, and negotiated reconstruction. Formally, for a sequence of uses $t = 1, 2, \ldots$, a slow noun tends toward stabilisation when*

$$d(B_{w_{t+1}}, B_{w_t}) \to 0 \tag{1.52}$$

under an appropriate semantic distance d, while never becoming perfectly fixed.

Examples in the present framework include *Alphon*, *Generon*, *Spherical Uncertainty Distribution*, *Geofinitism*, *containership*, and *Alphonic Pi*. These terms are not quick labels. They are slow containment objects.

1.10 Results: Shannon and Geofinite Information Compared

1.10.1 Summary Table

Category	Shannon Treatment	Geofinite Treatment
Primitive object	Selectable message or symbol	Finite symbolic containment
Symbol geometry	Absent/dimensionless	Explicit finite containment $C_i \subset \mathbb{R}^3$
Meaning	Excluded from engineering problem	Treated as containment/reconstruction, not perfect transfer
Base	Unit choice for logarithm	Structural property of symbolic measurement
Bit	Unit from base-2 logarithm	Projection of containment into binary distinction
Entropy	Uncertainty over symbol selection	Also path-loss after generonic instantiation
Compression	Removal of redundancy in symbol sequence	Projection that may discard geometry and provenance
Probability	Distribution over selectable alternatives	May be retained, but over finite containment objects
Geometry	Not part of information measure	Primary at Alphonic Limit
Circle closure	Not addressed as information problem	Requires minimum symbolic containment; EAP gives $B_{\min} = 4$
Base invariance	Holds up to unit conversion	Fails generally for finite symbolic geometry

1.10.2 Result 1: Shannon Information is a Projection

The Shannon object $(\mathcal{A}, p)$ can be obtained from the Geofinite object

$$\mathcal{I}_G = (\mathcal{A}_B^G, p, C, \Gamma_{\alpha,B}, \alpha) \tag{1.53}$$

by projection:

$$\Pi_S(\mathcal{I}_G) = (\mathcal{A}_B, p). \tag{1.54}$$

Therefore Shannon information may be understood, in this meta-model, as a projected theory of distinguishability after finite symbolic geometry has been removed.

1.10.3 Result 2: Base Invariance Splits

In conventional arithmetic and Shannon information, base change is a change of representation or unit. In Geofinite information, base changes the containment structure. Therefore:

$$V_B(s_B) = V_C(s_C) \tag{1.55}$$

does not imply

$$C_B(s_B) = C_C(s_C). \tag{1.56}$$

The abstract value may remain invariant while the symbolic geometry changes.

1.10.4 Result 3: Base 4 is the First EAP Closure Threshold

The EAP construction gives

$$B_{\min} = \lceil 10/\pi \rceil = 4. \tag{1.57}$$

This provides a concrete example where binary distinction is insufficient for planar circular closure.

1.10.5 Result 4: Not All Difference is Geometry

The base-4 threshold suggests:

> Not all measurable differences are geometric, and not all geometric properties are reducible to distinction.

This is a central conceptual result. A binary system may distinguish, but it does not necessarily contain closure, orientation, or return.

1.10.6 Result 5: Entropy Can Be Re-read as Path Loss

Shannon entropy measures uncertainty over already-instantiated symbols. The Geofinite formalism introduces a prior loss:

$$H(\Omega \mid S), \tag{1.58}$$

which measures, when probabilistically defined, the non-recoverable path uncertainty after the generonic map has produced finite symbols.

This suggests a wider conjecture:

Conjecture 1.1 (Entropy as Generonic Path Loss). *All entropy may be interpreted, at an appropriate level of description, as non-recoverable path-loss under finite symbol instantiation and projection.*

This conjecture is intentionally broad and requires substantial future work.

1.11 Discussion

1.11.1 What Shannon Made Possible

Shannon's abstraction succeeded because it removed difficult questions. By excluding semantic content and symbol geometry, it produced a universal engineering theory of communication. This allowed noise, coding, redundancy, compression, channel capacity, and transmission efficiency to be treated mathematically.

This chapter does not challenge that success. Rather, it identifies the cost of the abstraction. Shannon's symbol is powerful because it is flattened. It is selectable, countable, transmissible, and compressible precisely because its containment geometry is absent.

1.11.2 The Cost of Flattening

Flattening is not neutral. When a theory removes geometry, meaning, provenance, and path in order to become mathematically tractable, later theories that inherit the abstraction may become constrained by what was removed.

If the only admissible language is bits, states, amplitudes, probabilities, channels, and codes, then questions will be shaped by those containers. The projection may become a self-reinforcing framework. It works powerfully within its own geometry, but may make other geometries difficult to express.

This is especially important when information-theoretic language is extended into physics, biology, cognition, artificial intelligence, and interpretation of measurement.

1.11.3 Quantum Theory and Projected Probability

The Geofinite position is not that quantum mechanics is wrong. It is that Hilbert-space probability may be understood as a projected symbolic management of uncertainty rather than as the primary containment geometry of measurement.

A qubit in $\mathbb{C}^2$ is not a simple classical bit. It has complex amplitudes, phase, superposition, and measurement probabilities. Nevertheless, it is still represented in an abstract state space whose relation to finite geometric containership is not explicit.

In the present meta-model, this helps explain why quantum mechanics can be operationally successful while remaining interpretively difficult. It predicts outcome probabilities, but it does not necessarily state what finite geometric containment the information pertains to before projection.

1.11.4 The Sphere and the Cube

At the Alphonic Limit, the sphere represents isotropic uncertainty. The cube represents orthogonal addressability. Both are necessary in different ways.

The sphere says: below this limit, no direction can be privileged.

The cube says: to measure, one needs addressable distinctions.

The tension between them produces root-two and root-three constraints. Diagonal distance is not edge distance. Spherical containment and orthogonal address are not the same geometry.

This is not a defect. It is a clue that measurement contains both uncertainty and addressability, and that no single flattened symbol can preserve all geometry without loss.

1.11.5 From Planar Closure to Spherical Containment

The EAP result concerns planar circular closure. The next question is spherical containment.

If a circle requires at least four orientational distinctions for planar closure, what is the corresponding threshold for spherical enclosure?

Possible candidates include:

$$6 : \text{the six signed axial directions } \{\pm x, \pm y, \pm z\}, \tag{1.59}$$

$$8 : \text{the eight octants of a cube,} \tag{1.60}$$

$$12 : \text{edge-adjacency structures or richer packing relations.} \tag{1.61}$$

These are not yet results. They are research directions.

A future Spherical Alphonic Condition may take the form

$$B \geq B_{\text{sphere}}, \tag{1.62}$$

where B_{sphere} is the minimum symbolic base or containment cardinality required to communicate spherical enclosure without collapsing it into a lower-dimensional projection.

1.11.6 Language, Slow Nouns, and Mathematical Containment

The present work also clarifies why language is not external to mathematics. Mathematics is a constrained symbolic language. If symbols are finite containment objects, then mathematical language is part of the measurement process by which relations become communicable.

The reader is the author in the sense that the reader reconstructs the containment basin. A formal expression does not transmit perfect meaning. It constrains reconstruction.

This is why slow nouns are necessary in new theoretical work. New words such as *Alphon* or *Generon* cannot be fully defined in a single sentence. They require repeated symbolic negotiation until their containment geometry stabilises sufficiently for use.

1.12 Future Directions

1.12.1 A Geofinite Source Coding Theorem

A future theorem may attempt to extend Shannon source coding to finite geometric symbols. Instead of minimising expected code length alone, one would minimise expected code length subject to containment preservation:

$$\min_{\mathcal{E}} \mathbb{E}[\ell(\mathcal{E}(S))] \quad \text{subject to} \quad D_G(C_S, C_{\hat{S}}) \le \varepsilon, \tag{1.63}$$

where $\mathcal{E}$ is an encoding, ℓ is code length, D_G is a geometric containment distortion, and ε is an admissible loss threshold.

This would generalise compression from symbol-sequence recovery to containment-geometry recovery.

1.12.2 Path-Loss Entropy

A formal path-loss entropy requires a measure on generative path space Ω. If such a measure can be defined, then

$$H_G = H(S) + \lambda H(\Omega \mid S) \tag{1.64}$$

may serve as a first candidate for a combined symbolic and generonic entropy, where λ controls the weight of path-loss relative to symbol uncertainty.

This is only a placeholder. A stronger treatment would avoid arbitrary weighting and derive the relation from measurement constraints.

1.12.3 Base-Dependent Geometry Experiments

The EAP generator suggests practical experiments:

1. compute EAP across bases and alphabets;
2. test closure thresholds for circular, spherical, and toroidal relations;
3. compare binary, quaternary, octal, decimal, hexadecimal, base-36, and nonstandard alphons;
4. evaluate whether geometric error changes structurally with base;
5. study whether higher bases preserve containment features lost under binary projection.

1.12.4 Spherical Packing and Fractal Containment

The Spherical Uncertainty Distribution suggests that measurement may involve nested or packed uncertainty spheres rather than point distributions. Future work should investigate:

- sphere packing at the Alphonic Limit;
- flattened circular distributions as projections of spherical packings;
- fractal containment structures under repeated generonic instantiation;
- links between packing density, uncertainty, and symbolic resolution.

1.12.5 Quantum Reinterpretation

A cautious research direction is to compare Hilbert-space probability distributions with flattened spherical uncertainty distributions. The question is not whether standard quantum predictions can be reproduced immediately. The first question is whether a finite containment model can explain why the Hilbert-space projection is so effective while also clarifying what geometric content it omits.

1.12.6 Language and Artificial Intelligence

Large language models operate over finite token sequences. If language is a nonlinear dynamical system and words are containment basins, then token generation may be studied as a trajectory through symbolic containment space rather than merely as next-token prediction.

This suggests possible links to:

- semantic phase-space reconstruction;
- slow noun stabilisation;
- path-loss in tokenisation;
- embedding compression and geometric loss;
- safety risks arising from distorted symbolic containment.

1.13 Conclusion

Shannon information theory treats information as uncertainty over selectable alternatives. This abstraction made modern communication theory possible. Its symbols are dimensionless by design.

The Geofinite framework developed here asks what happens when this design choice is reversed. If symbols are finite geometric containment objects, then base is no longer merely a unit choice, entropy may include generonic path-loss, binary distinction becomes a projection rather than a primitive geometry, and base invariance splits into abstract value invariance and geometric non-invariance.

The Equivalent Alphonic Pi result provides a concrete hinge. The Alphonic Circle Condition gives

$$B_{\min} = \lceil 10/\pi \rceil = 4, \tag{1.65}$$

suggesting that planar circular closure requires a minimum symbolic containment beyond binary distinction.

The wider principle is simple but significant:

> Measurability is not solely a matter of precision. It is also a matter of finite symbolic structure.

Or, more directly:

> A symbol does not merely distinguish. A symbol contains.

Chapter 2

Functional Symbolic Trajectories and Metrological Anchoring

Status of this Chapter

This chapter is a working research draft. It is intended to follow the chapter on Geofinite information, symbolic containership, and the Alphonic Circle Condition. The previous chapter developed the claim that a finite symbol does not merely distinguish; it contains. The present chapter develops the complementary claim that a finite symbol does not merely contain; it moves.

The central term introduced here is the *functional symbolic trajectory.* This term is proposed as a gateway concept for readers entering the basin of Geofinitism and Finite Symbolic Mechanics. It is intended to be accessible enough for use in ordinary exposition while remaining formal enough to connect with symbolic containership, the Generonic Boundary, the Alphonic Limit, semantic phase space, language as a nonlinear dynamical system, and metrological practice.

The chapter does not claim that Geofinitism is proven. Geofinitism is treated as a model-trajectory with error bounds relative to other symbolic trajectories. It is a proposed way of organising finite symbolic measurement, not a privileged position outside language. The chapter therefore applies its own method to itself: the term *functional symbolic trajectory* is also a slow noun, introduced here as a stabilising symbolic pathway whose meaning must be refined through repeated use.

2.1 Motivation: From Symbolic Containment to Symbolic Movement

The preceding chapter introduced the Geofinite treatment of information as finite symbolic containership. In that treatment, a finite symbol is not a dimensionless label. It is a distinguishable mark together with a finite containment geometry. The symbol is therefore not exhausted by its visual or formal mark. It carries a geometry of uncertainty, a relation to a measurement frame, and a non-recoverable generative path.

That construction is necessary, but not sufficient. A symbol is not normally encountered as an isolated container. It appears in use. It is read, repeated, transformed, stabilised, compressed, extended, translated, formalised, disputed, and sometimes allowed to fail. A word in an article, a number in a metrological definition, an equation in a physical theory, a noun in a philosophical argument, or a token in a language model is not merely a static object. It participates in a movement through symbolic space.

This movement is what this chapter calls a *functional symbolic trajectory.*

The need for this term becomes clear when examining scientific and philosophical exposition.

An article may begin with a mathematically constrained claim, move through analogy, pass into narrative projection, and end by suggesting a future world-picture. A physical term such as *photon* may move between experiment, quantum field theory, pedagogical metaphor, ordinary language, and personal interpretation. A metrological definition such as the SI second may begin in an atomic transition, become a counted number of periods, enter an international convention, and then propagate through instruments, textbooks, software, and later theoretical models.

In each case, the symbol does not remain a fixed thing. It becomes a finite symbolic pathway that carries representation, constraint, and uncertainty through time and use.

2.2 Definition of a Functional Symbolic Trajectory

Definition 2.1 (Functional Symbolic Trajectory)**.** *A* functional symbolic trajectory *is a finite symbolic pathway that carries representation, constraint, and uncertainty through words, mathematics, measurement, memory, and social use. It is not a thing, but a stabilised movement that can be followed, tested, compressed, extended, perturbed, reconstructed, or allowed to fail.*

This definition is intentionally broad. It is meant to apply to everyday words, technical nouns, mathematical expressions, metrological constants, scientific theories, philosophical arguments, and token sequences in artificial language systems. The definition should not be read as a final metaphysical statement. It is a working containment for a class of symbolic phenomena that are usually treated as static.

Three components are central.

- **Representation** describes how the trajectory fits into the wider semantic extent or semantic phase space of trajectories.
- **Constraint** describes what binds, limits, or shapes the trajectory, including measurement, grammar, mathematical rules, instrumentation, prior use, social agreement, and rules of admissibility.
- **Uncertainty** describes the local region or neighbourhood of possible neighbouring trajectories surrounding the current symbolic trajectory.

The term *expectation* may still be used informally, but it is not taken as primitive. Expectation is treated here as a projected or readable surface of local uncertainty. In a language model, for example, the next-token distribution is not a human-like expectation. It is a finite projection from a local uncertainty region into a symbolic selection space.

2.3 A Preliminary Formal Representation

Let M denote a semantic phase space. This need not initially be assumed to be a smooth manifold in the strict mathematical sense. It denotes the wider symbolic extent in which words, marks, equations, models, and meanings are reconstructed as finite trajectories. Let t index stages of use, reading, measurement, computation, or exposition.

A functional symbolic trajectory may be represented schematically as

$$\mathcal{T}_s(t) = (\Sigma_t, R_t, K_t, U_t, H_t)\,, \tag{2.1}$$

where

- $\Sigma_t = (\sigma_t, C_t)$ is the finite symbol at stage t, consisting of a distinguishable mark σ_t and finite containment geometry C_t;

- R_t is the representational relation of the symbol to the wider semantic phase space M;
- K_t is the constraint or admissibility structure acting on the trajectory at stage t;
- $U_t \subseteq M$ is the local uncertainty region surrounding the current trajectory;
- H_t is the history, provenance, or prior path through which the trajectory has been stabilised.

The trajectory itself is then the finite sequence or path

$$\mathcal{T}_s : \mathcal{T}_s(t_0) \to \mathcal{T}_s(t_1) \to \cdots \to \mathcal{T}_s(t_n). \tag{2.2}$$

This expression is not intended to imply that all components are always known, measurable, or explicitly recoverable. Indeed, one of the central points of the Geofinite framework is that provenance and generative path are often partially lost after finite symbol instantiation. The purpose of the notation is to make explicit that a symbol-in-use carries more than a mark.

2.4 Representation, Constraint, and Local Uncertainty

2.4.1 Representation

Representation is the way a trajectory fits into the wider symbolic space of other trajectories. A word such as *particle* has one representational role in ordinary language, another in classical mechanics, another in quantum field theory, and another in public science writing. It is not the same functional symbolic trajectory in each case, even when the visible word remains unchanged.

Formally, let $\mathcal{B}(\mathcal{T}_s) \subseteq M$ denote the semantic containment basin associated with a trajectory. A reconstruction by reader, instrument, model, or community r may be written

$$M_r(\mathcal{T}_s) = R_r\left(\mathcal{B}(\mathcal{T}_s)\right), \tag{2.3}$$

where R_r is the reconstruction map associated with that reader or system. This generalises the earlier claim that a word is not merely transmitted. It is reconstructed.

Meaning may then be described as the measured representational fit of a trajectory within the wider semantic phase space. In this sense, measurement and meaning are not wholly separate. Meaning is not a hidden substance inside a word. It is a finite relational placement within a manifold of possible and prior trajectories.

2.4.2 Constraint

Constraint is what prevents symbolic movement from becoming arbitrary. A mathematical proof is constrained by formal rules. A metrological definition is constrained by an instrumentally reproducible measurement practice. A scientific model is constrained by observation, convention, calibration, and predictive success. A social term is constrained by repeated use and negotiated agreement.

Let K_t denote the set of admissibility conditions acting at stage t. The locally admissible region of the trajectory may be written

$$A_t = U_t \cap K_t, \tag{2.4}$$

where U_t is the local uncertainty region and K_t is treated as a constraint structure. More explicitly, if K_t is represented as a predicate over possible continuations $\tau \in U_t$, then

$$A_t = \{\tau \in U_t : K_t(\tau) = 1\}. \tag{2.5}$$

A trajectory slips when it moves from one constraint regime into another while continuing to borrow authority from the earlier regime. This is common in scientific exposition. A text may begin with formal mathematics, then move into analogy, then into speculative projection, while preserving the rhetorical force of the initial mathematical constraint.

2.4.3 Local Uncertainty

Uncertainty is the local region of possible neighbouring trajectories surrounding the present symbolic state. This is especially important for language. A sentence does not determine a single perfect reconstruction. It opens a neighbourhood of possible reconstructions. Some are close to the intended trajectory. Others drift, branch, compress, or enter another attractor.

If a semantic distance d_M can be defined, a simple local uncertainty region may be written

$$U_t(\epsilon) = \{m \in M : d_M(m, \mathcal{T}_s(t)) \leq \epsilon\}. \tag{2.6}$$

In many cases, such a metric will not yet be available. Then U_t should be understood more generally as a neighbourhood or local manifold of possible continuations and reconstructions. The important point is that uncertainty is not an external defect added after meaning. It is part of the finite symbolic condition.

For a language model, the projected token distribution may be represented schematically as

$$P(x_{t+1} \mid x_{\leq t}) = \Pi_{\text{tok}}(U_t, K_t), \tag{2.7}$$

where Π_{tok} projects the local uncertainty region and constraint structure into a finite distribution over next-token candidates. This is not a claim that the language model possesses expectation in a human sense. It is a claim that the observable next-token distribution may be read as a finite projection of local symbolic uncertainty.

2.5 Functional Symbolic Trajectories and the Generonic Boundary

The previous chapter introduced the Generonic Boundary as a map from pre-symbolic interaction or generative path space Ω into finite symbolic strings. In that construction, the map

$$\Gamma_{\alpha,B} : \Omega \to A_B^{G*} \tag{2.8}$$

is many-to-one. A finite symbol string s does not generally preserve its full originating path. The fibre

$$\Gamma_{\alpha,B}^{-1}(s) \tag{2.9}$$

contains the set of pre-symbolic paths that instantiate as s. Unless that fibre is a singleton, the full path is not recoverable.

A functional symbolic trajectory begins after, or across, this generonic act. It is the movement of instantiated finite symbols through later symbolic systems. It therefore inherits generonic path-loss. The trajectory can be stabilised, but not made perfect. It can be constrained, but not made omniscient. It can be measured, but always from within a finite symbolic frame.

This gives a direct connection between symbolic containership and symbolic movement:

$$\Omega \xrightarrow{\Gamma_{\alpha,B}} S \xrightarrow{\mathcal{T}} M. \tag{2.10}$$

The generonic map instantiates finite symbols. The trajectory map carries those symbols through semantic phase space. The first transition loses generative path. The second transition accumulates history, constraint, representation, and further uncertainty.

2.6 Slow Nouns as Stabilising Trajectories

New theoretical work often requires new nouns. Such nouns cannot be fully stabilised by a single definition. They require repeated use, refinement, correction, and negotiated reconstruction. The previous chapter called these *slow nouns*. In the present chapter, a slow noun may be understood as a functional symbolic trajectory whose basin stabilises gradually.

Let w_t denote a word or symbolic expression across repeated uses. Let $\mathcal{B}_{w_t}$ denote its semantic containment basin at stage t. A slow noun tends toward local stabilisation when

$$d(\mathcal{B}_{w_{t+1}}, \mathcal{B}_{w_t}) \to 0 \tag{2.11}$$

under an appropriate semantic distance d, while never becoming perfectly fixed.

The phrase *functional symbolic trajectory* is itself a slow noun. So are *Alphon*, *Generon*, *Alphonic Limit*, *Spherical Uncertainty Distribution*, *containership*, and *Geofinitism*. Their purpose is not to produce final verbal closure at first use. Their purpose is to make a new region of symbolic movement communicable.

This is one reason why beginning with functional symbolic trajectories may help readers enter the basin of Geofinitism. The phrase starts from a familiar experience: a sentence moves, an article carries a reader, a theory develops, a word changes across contexts. From there, the reader can be guided toward nonlinear dynamics, semantic phase space, attractor basins, generonic path-loss, and finite symbolic measurement.

2.7 Metrological Functional Symbolic Trajectories

A special case of a functional symbolic trajectory occurs when the trajectory is anchored by formal measurement practice.

Definition 2.2 (Metrological Functional Symbolic Trajectory)**.** *A* metrological functional symbolic trajectory *is a functional symbolic trajectory whose stabilisation depends on a formal measurement definition, instrumentally reproducible procedure, conventional unit, calibration chain, or recognised metrological authority.*

More simply, this may be called a *metrologically anchored symbolic trajectory*.

This class of trajectory is important because it shows that measurement definitions are not merely static statements. They are operational-symbolic pathways. A metrological definition begins in a chosen physical relation or reproducible procedure, enters a formal symbolic convention, becomes embedded in instruments, and then propagates into scientific language, calculation, software, education, and model-building.

A metrological symbolic trajectory may be represented as

$$\mathcal{T}_{\text{met}} : P \to C_m \to D \to I \to L \to M, \tag{2.12}$$

where

- P is the physical process or relation selected for measurement;
- C_m is the counted or measured relation;
- D is the formal definition;
- I is the instrument or calibration infrastructure;
- L is the linguistic and symbolic representation;
- M is the later model space into which the definition propagates.

This representation is deliberately schematic. The point is that a unit definition is not merely a sentence. It is a trajectory through physical process, symbolic convention, institutional agreement, instrumentation, and later semantic inheritance.

2.8 Worked Example: The SI Second as a Metrological Trajectory

The SI second provides a useful example. In contemporary metrology, the second is defined through a specified transition of the caesium-133 atom. The definition fixes a precise integer number of periods of the radiation corresponding to that transition. Operationally, this gives physics a highly stable counting anchor.

The standard reading is that the integer count is an abstract value. It can be written in decimal, binary, hexadecimal, base twelve, or any other base without changing the underlying count. This is correct within ordinary numerical base invariance.

The Geofinite reading does not deny this. It adds a further distinction. Once the count enters symbolic use, it is represented in a finite base. That represented number is not merely an abstract value. It is a finite symbolic trajectory. It is written, memorised, taught, tokenised, processed, stored, printed, transmitted, and inherited by later models.

Thus the SI second may be written as a metrological functional symbolic trajectory:

$$\mathcal{T}_{\mathrm{Cs}} : \text{atomic transition} \to \text{period count} \to \text{SI definition} \to \text{decimal inscription} \to \text{instrumental realisation} \to \text{sc} \tag{2.13}$$

The important distinction is this:

> The physical count may be abstractly base-invariant, but the represented count is not trajectory-invariant.

This sentence captures the Geofinite concern. The claim is not that the physical transition changes when the number is written in another base. The claim is that finite symbolic representation changes the trajectory through which the count is stored, processed, remembered, compressed, reconstructed, and extended.

In conventional metrology, this representational difference is usually negligible. For engineering and laboratory practice, that is appropriate. But in Geofinitism and Finite Symbolic Mechanics, finite symbolic geometry is not generally neutral. If base has geometry, then a base-ten metrological inscription may constrain downstream semantic geometry in ways that are invisible to a theory that treats representation as a transparent label.

2.9 Base Invariance and Trajectory Non-Invariance

The previous chapter introduced the principle of split base invariance. Abstract numerical value may be invariant under base conversion, but finite symbolic geometry is not generally invariant. The present chapter extends that idea from symbolic containment to symbolic trajectory.

Let x be an abstract numerical value, and let $R_B(x) = s_B$ denote its representation in base B. In ordinary arithmetic,

$$V_B(s_B) = V_C(s_C) = x, \tag{2.14}$$

where $s_C = R_C(x)$ is the representation of the same abstract value in base C.

In the Geofinite treatment, however, each representation has a containment geometry and a trajectory history:

$$\mathcal{T}_{s_B} \neq \mathcal{T}_{s_C} \quad \text{in general.} \tag{2.15}$$

The same abstract value may therefore generate different finite symbolic trajectories under different bases:

$$V_B(s_B) = V_C(s_C) \Rightarrow \mathcal{T}_{s_B} = \mathcal{T}_{s_C}. \tag{2.16}$$

This is the principle of *trajectory non-invariance.*

Principle 2.1 (Trajectory Non-Invariance)**.** *Abstract value invariance under base conversion does not imply invariance of finite symbolic trajectory. When symbols possess finite containment geometry, different base representations may preserve abstract value while altering symbolic length, compression, visual form, tokenisation, memory, computational handling, and semantic inheritance.*

This principle is especially important when a represented number becomes a metrological anchor. A base-ten integer used in a foundational definition is not merely a neutral display. It is part of the symbolic pathway by which later models inherit the measurement.

2.10 Application: Photon Language as a Slipping Trajectory

The term *photon* provides a useful example of symbolic trajectory instability. In public and semi-technical writing, the photon is often described as a particle of light. This language is pedagogically useful, but it can mislead if the word *particle* is reconstructed through ordinary material-body expectations. The technical term does not refer to a small classical object with rest mass and ordinary three-dimensional locality.

A common corrective move is to reject the word *particle* and replace it with a more intuitive physical picture, such as a wave-cycle. Such a move may begin from a legitimate discomfort with public language, but it can slip if it does not remain constrained by the mathematics and metrology it invokes.

For example, the relation

$$E = h\nu \tag{2.17}$$

binds photon energy to frequency through Planck's constant. A trajectory can slip if h is interpreted as the energy of every single electromagnetic cycle independent of frequency, and if the photon is then fixed as one complete wave cycle. The symbolic discomfort around *photon as particle* is replaced by another symbolic compression: *photon as one cycle.* The new noun feels more physical, but it does not remain admissible under the rule structure of the equation it invokes.

This example illustrates the value of functional symbolic trajectory analysis. The question is not first whether a public phrase is true or false. The question is how the trajectory moves:

$$\text{technical term} \to \text{ordinary noun} \to \text{semantic discomfort} \to \text{replacement image} \to \text{mathematical slippage} \to \text{ne} \tag{2.18}$$

The point is not to defend any inherited terminology. It is to locate the transition where a trajectory leaves one constraint regime and continues as if that regime still holds.

2.11 The Five-Pillar Diagnostic as Trajectory Measurement

A functional symbolic trajectory can be examined using a diagnostic method. One such method is the analysis of five pillars. The details of that method may be developed elsewhere, but its role in the present chapter is to measure where an exposition remains constrained and where it breaks down.

A preliminary five-pillar reading may ask:

1. What symbolic trajectory is being constructed?
2. Which words, numbers, equations, or diagrams act as stabilising pillars?
3. Which constraint regimes are being invoked: measurement, mathematics, analogy, authority, narrative, or social use?
4. Where does the trajectory introduce infinity, perfection, dimensionlessness, pointness, or continuity in order to instantiate a finite measurable value?
5. Does the trajectory return to an admissible measurement anchor, or does it continue as an unconstrained projection?

The fifth question is central. A trajectory need not be rejected because it contains projection or analogy. New theory often requires speculative movement. However, the movement should be marked. A mathematical trajectory should not quietly become a narrative trajectory while retaining the authority of mathematical closure. A metrological trajectory should not quietly become an ontology while hiding its symbolic representation. A linguistic trajectory should not pretend to transfer perfect meaning while relying on reconstruction by the reader.

In this sense, the five-pillar diagnostic is a measurement technique for symbolic exposition. It examines the stability of a trajectory relative to its declared and undeclared constraints.

2.12 Functional Symbolic Trajectories in Language and Artificial Intelligence

Language provides the most immediate example of functional symbolic trajectories. A sentence unfolds as a finite sequence of symbols. Each word carries a prior history and opens a local uncertainty region of possible continuations and reconstructions. Meaning is not transferred perfectly from writer to reader. The sentence constrains reconstruction.

For artificial language systems, the trajectory is directly observable in token sequence form. A language model receives a finite symbolic history $x_{\leq t}$ and produces a distribution over possible continuations. This may be read as a projected management of local uncertainty rather than as direct access to meaning.

A simplified language-model trajectory may be written

$$\mathcal{T}_{\text{LLM}} : x_1 \to x_2 \to \cdots \to x_t \to P(x_{t+1} \mid x_{\leq t}). \tag{2.19}$$

From the Geofinite perspective, the token distribution is not the full semantic geometry. It is a finite projection from a local uncertainty region in an embedding or semantic phase space into a discrete token alphabet. Tokenisation, embedding, attention, compression, and decoding all shape the trajectory.

This suggests that large language models may be studied as systems that reconstruct and extend functional symbolic trajectories. Their apparent fluency arises not from possession of fixed meanings, but from movement through stabilised regions of symbolic phase space. Their failures may therefore be studied as trajectory failures: collapse into attractors, loss of constraint, distorted containment, excessive compression, or unstable reconstruction.

This also explains why slow nouns are difficult for such systems. New theoretical terms have not yet stabilised across wide public language. Their containment basins are narrow, emerging, and locally negotiated. A model may therefore pull them toward nearby established attractors unless the local trajectory is strongly constrained by context.

2.13 Results

The present chapter proposes the following preliminary results.

2.13.1 Result 1: Symbols Become Meaningful Through Trajectory

A finite symbol is not exhausted by its mark or containment geometry. It becomes meaningful through use, reconstruction, constraint, history, and local uncertainty. Meaning may therefore be treated as measured representational fit within semantic phase space.

2.13.2 Result 2: Containment and Trajectory Are Complementary

The previous chapter argued that a symbol contains. The present chapter argues that a symbol moves. Symbolic containership describes the finite geometric condition of the symbol. Functional symbolic trajectory describes the symbol's stabilised movement through use.

2.13.3 Result 3: Expectation Is Not Primitive

Expectation should not be treated as a primitive in the formalism. The more basic object is local uncertainty: the neighbourhood of possible continuations, reconstructions, and adjacent trajectories surrounding the current symbolic state. Expectation is a projection from that uncertainty under a particular system of reconstruction.

2.13.4 Result 4: Metrological Definitions Are Symbolic Trajectories

A metrological definition is not merely a static convention. It is a metrological functional symbolic trajectory linking physical process, counted relation, formal definition, instrumental practice, symbolic inscription, and later model-building.

2.13.5 Result 5: Abstract Base Invariance Does Not Imply Trajectory Invariance

An abstract numerical value may be preserved across base conversion, but the finite symbolic trajectory of its representation is not generally preserved. This matters when a represented number becomes a metrological anchor or a repeated component of scientific language.

2.13.6 Result 6: Expository Slippage Can Be Measured

Scientific and philosophical exposition can be analysed by locating where a trajectory moves between measurement, mathematics, analogy, narrative, and projection. Slippage occurs when authority from one constraint regime is carried into another without acknowledgement.

2.14 Discussion

2.14.1 Why the Term Is Useful

The term *functional symbolic trajectory* is useful because it provides a reader-friendly gateway into the deeper structure of Geofinitism and Finite Symbolic Mechanics. It begins with a familiar observation: words, sentences, articles, theories, and equations move. They do not sit still as perfect objects. They unfold through time, use, reconstruction, and constraint.

From this starting point, the reader can be guided toward more technical concepts. A trajectory suggests nonlinear dynamics. A symbolic trajectory suggests language and mathematics. A functional symbolic trajectory suggests that the movement performs work. It carries representation, constraint, and uncertainty. It can succeed, drift, bifurcate, compress, or fail.

This avoids beginning with more difficult terms such as generonic boundary, alphonic limit, semantic phase space, or spherical uncertainty distribution. Those terms remain necessary, but the reader can enter the basin through a more ordinary experience of symbolic movement.

2.14.2 Relation to Geofinitism

Geofinitism is not treated here as an external truth against which all other theories are judged. It is itself a functional symbolic trajectory. Its terms are slow nouns. Its commitments are rules of admissibility. Its claims carry uncertainty. Its value lies in whether it provides a useful, coherent, and measurable way to organise symbolic, mathematical, and physical relations.

This self-application is important. A theory of symbolic trajectories must not exempt itself from symbolic trajectory analysis. The present framework therefore treats its own vocabulary as provisional, finite, and subject to refinement.

2.14.3 Relation to Mathematics

Mathematics may be treated as a highly constrained class of functional symbolic trajectories. A mathematical expression is not merely a mark. It belongs to a rule-governed pathway. The pathway may be followed, checked, transformed, and extended under formal constraints.

However, mathematical exposition can slip when it leaves formal rules and continues as narrative projection. This does not make such projection illegitimate. It means the transition should be marked. A conjecture, interpretation, analogy, or future direction is not the same type of trajectory as a proof.

2.14.4 Relation to Metrology

Metrology provides one of the clearest examples of symbolic anchoring. Units are not merely convenient labels. They stabilise relations between physical processes, instruments, definitions, and symbolic systems. Once a metrological definition enters language, it becomes part of the semantic geometry of later scientific work.

The SI second example shows how a counted physical relation becomes a symbolic anchor. In standard practice, the base in which the count is written is not operationally central. In the Geofinite treatment, however, the base representation contributes to the finite symbolic trajectory by which the measurement is inherited.

2.14.5 Relation to Artificial Intelligence

Artificial language systems make symbolic trajectory visible. They operate over finite token sequences and project local uncertainty into continuation distributions. This makes them useful experimental systems for studying language as a nonlinear dynamical process.

If words are symbolic containment basins and sentences are trajectories through semantic phase space, then language-model behaviour can be studied in terms of trajectory stability, attractor drift, compression loss, tokenisation path-loss, and reconstruction uncertainty. This may also provide language for understanding safety risks arising from distorted embeddings or corrupted symbolic containment.

2.15 Future Directions

Several directions follow from this chapter.

2.15.1 Formal Metrics for Semantic Trajectory Distance

The formalism requires better definitions of semantic distance. In language models, embedding-space distance, attention patterns, activation trajectories, and token-distribution divergence may provide practical approximations. In human language, semantic distance may require a combination of textual, historical, social, and metrological reconstruction.

2.15.2 Trajectory-Based Analysis of Scientific Articles

Public scientific articles, essays, and theoretical papers can be analysed as functional symbolic trajectories. This would allow one to identify where exposition remains measurement-bound, where it becomes mathematical, where it shifts into analogy, and where it projects a future framework.

2.15.3 Metrological Trajectory Mapping

A systematic study could map major SI definitions as metrological functional symbolic trajectories. Each unit could be examined in terms of physical anchor, symbolic inscription, base representation, instrumentation, calibration, and downstream semantic inheritance.

2.15.4 Base-Dependent Symbolic Geometry

The principle of trajectory non-invariance suggests experiments comparing how different base representations alter symbolic length, compression, tokenisation, human recall, machine processing, and geometric interpretation. This may connect the earlier Equivalent Alphonic Pi construction to practical symbolic systems.

2.15.5 Language-Model Experiments

Functional symbolic trajectories may be studied directly in language models by perturbing prompts, tokenisations, embeddings, or compression levels and observing changes in continuation trajectories. This may provide empirical access to local uncertainty regions and attractor behaviour in symbolic phase space.

2.16 Conclusion

The previous chapter argued that a symbol does not merely distinguish. A symbol contains. The present chapter adds that a symbol does not merely contain. A symbol moves.

A functional symbolic trajectory is the finite symbolic pathway by which a mark, word, number, equation, model, or theory carries representation, constraint, and uncertainty through use. It is not a thing. It is a stabilised movement through symbolic phase space.

A metrological functional symbolic trajectory is a special case in which that movement is anchored by measurement practice. The SI second illustrates this structure. A physical transition becomes a counted relation, the counted relation becomes a formal definition, the definition becomes an instrumental practice, and the symbolic representation becomes part of later scientific language.

This matters because abstract invariance does not guarantee trajectory invariance. The same numerical value may be preserved under base conversion, but its finite symbolic pathway may change. If base has geometry, then representation is not neutral.

The wider principle may be stated simply:

> A symbol contains. A symbol moves. A measured symbol anchors. A repeated symbol becomes a trajectory. A constrained trajectory becomes meaning.

This chapter therefore provides a gateway into Geofinitism and Finite Symbolic Mechanics. It invites the reader to stop treating words, numbers, and theories as fixed things, and to begin examining how they move, stabilise, slip, and return to measurement within the manifold of meaning.

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