The Attralucian Essays:

Exploring the Finite

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Dissolution of the Riemann Hypothesis: A Phase-Space Reconstruction Approach

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Overview

The Riemann Hypothesis, traditionally formulated under Assumption PC (Platonic Continuum), posits that nontrivial zeros of the zeta function lie exactly on the critical line $\mathrm{Re}(s) = 1/2$. This claim assumes base-invariant truth in the completed infinite field $\mathbb{C}$. We adopt instead Assumption GF (Geofinitist Finite), which recognizes that mathematical computation occurs through finite symbolic operations with measurement constraints. Under GF, we demonstrate that the observed clustering of zeros near $\mathrm{Re}(s) = 1/2$ emerges naturally from viewing $\zeta(s)$ through the lens of phase-space dynamics. By establishing the operational equivalence between Takens delay embedding and complex plane representation, we show that the critical line corresponds to the geometric centre of base-10 computational space—the phase-space attractor for prime distribution dynamics. This provides a *Geofinitist Resolution*: a geometric explanation of why the pattern must occur within finite measurement frameworks, without claiming base-invariant exactness or infinite verification. This approach reveals that the critical line location is measurement-system-dependent, opening new directions for understanding the relationship between computational substrate and mathematical results. Our resolution exemplifies how many classical mathematical "proofs" may be reframed as Geofinitist Resolutions once we acknowledge the measurement foundations and symbolic provenance that standard proofs typically omit.

Keywords: Riemann Hypothesis, Phase-Space Embedding, Takens Theorem, Dynamical Systems, Zeta Function, Attractor Geometry, Computational Mathematics, Geofinitism, Measurement Foundations

Introduction: Foundations and Frameworks

The Measurement Foundations of Mathematics

Before formal mathematics existed, there was measurement. The first mathematical symbols emerged from the need to encode and communicate measurements—initially through spoken sounds (acoustic patterns), later through written marks (geometric forms). This progression from acoustic to symbolic is not merely historical curiosity; it reveals the fundamental nature of mathematical practice.

Mathematical notation is not a representation of Platonic ideals existing in some abstract realm, but rather a compression of measurement patterns into transmissible symbolic form. When we write equations like $F = ma$ or $\int f(x)\,dx$, we are encoding the results of countless physical measurements and logical operations into compact visual symbols. These symbols are descendants of acoustic reasoning—verbal arguments about quantities and relationships, compressed over centuries into the elegant notation we use today.

This view naturally extends to computation itself. Every mathematical calculation, from a simple sum to the evaluation of a complex integral, is a dynamical process operating within a finite state space. The trajectory of computational states—from the initial symbolic representation, through a sequence of algorithmic transformations, to the final output—constitutes a physical process whose emergent, stable patterns we interpret as mathematical results. Applied mathematics, in this light, is the study of these dynamical processes and their attractors.

Each mathematical symbol has three essential properties:

1. **Finite geometric form:** Every symbol occupies finite space, whether as ink on paper, pixels on screen, or acoustic wave in air.

2. **Provenance:** Every symbol has a history—who created it, when, in what context, for what purpose.

3. **Uncertainty bounds:** Every measurement and computation produces results with bounded precision.

Consequently, all computation operates within a measurement framework, not a Platonic realm of perfect forms. The choice of numerical base, precision limits, and operational structure of arithmetic all affect measurable outcomes. Every calculation ever performed has occurred in finite precision within a chosen symbolic system.

Implications for the Riemann Zeta Function

For the Riemann zeta function $\zeta(s)$, these considerations are crucial. Every computation of $\zeta(s)$ occurs in base-10 or base-2 arithmetic. The geometric structure of these bases affects where zeros appear in the complex plane.

The decimal system uses ten symbols $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ with midpoint 4.5. Normalized to the interval $[0, 1]$, this gives $4.5/9 = 0.5$. Thus, the critical line $\mathrm{Re}(s) = 1/2$ emerges geometrically from base-10 structure.

Two Frameworks: PC vs. GF

To clarify assumptions, we distinguish two foundational frameworks:

Assumption PC (Platonic Continuum):

- $\mathbb{C}$ treated as complete continuous field.
- Series like $\sum_{n=1}^{\infty} 1/n^s$ regarded as completed infinities.
- Analytic continuation and functional equations assumed exact.

- Base-invariance taken as axiomatic.

Assumption GF (Geofinitism):

- Mathematics as finite symbolic computation.
- Proofs as transductive symbolic transformations.
- Measurement-based: finite form, provenance, and uncertainty.
- Representation and base-dependence matter.

Under GF, the Riemann Hypothesis becomes: an observation that zeros cluster near $\mathrm{Re}(s) = 1/2$ in base-10 computation, explained geometrically.

The Geofinitist Resolution Concept

A Geofinitist Resolution differs from a classical proof. While PC proofs claim universal truth for infinite collections, GF resolutions explain finite, observable patterns within measurement limits.

For RH:

$$\begin{aligned} \text{Observed:} \quad & \text{Zeros cluster near } \mathrm{Re}(s) \approx 1/2. \\ \text{Explanation:} \quad & \text{This is the geometric centre of base-10 space.} \end{aligned}$$

Structure of this Paper

Sections 2–7 develop the argument:

- Section 2: Takens embedding and phase-space theory.
- Section 3: Equivalence of Takens and complex embedding.
- Section 4: Zeta as dynamical system.
- Section 5: Base-10 geometric resolution.
- Section 6: Implications, extensions, base-dependence.
- Section 7: Conclusions and epistemological implications.

Phase-Space Embedding: Mathematical Framework

Takens' Delay-Coordinate Embedding

Given a time series $\{x(t)\}$, the embedding of dimension m with delay τ is:

$$\mathbf{X}(t) = [x(t), x(t-\tau), x(t-2\tau), \dots, x(t-(m-1)\tau)].$$

For $m > 2d$, where d is the attractor dimension, this embedding is diffeomorphic to the original attractor.

For $m = 2$, e.g. $x(t) = \sin(\omega t)$ with $\tau = \pi/(2\omega)$, we obtain:

$$\mathbf{X}(t) = [\sin(\omega t), -\cos(\omega t)], \qquad x_1^2 + x_2^2 = 1,$$

a circular attractor.

Complex Plane as 2D Coordinate System

A complex number $z = x + iy$ represents $(x, y) \in \mathbb{R}^2$. Multiplication by i rotates coordinates by 90°, so $i^2 = -1$ expresses 180° rotation. Thus, complex numbers are geometric coordinates.

Hilbert Transform and Phase Shift

The Hilbert transform $\mathcal{H}x$ of $x(t)$ is:

$$\mathcal{H}x(t) = \frac{1}{\pi}\text{P.V.} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau.$$

It produces a 90° phase shift: $\mathcal{H}[\sin(\omega t)] = -\cos(\omega t)$, yielding analytic signal $z(t) = x(t) + i\mathcal{H}x(t) = e^{i\omega t}$.

Operational Equivalence: Takens and Complex Embedding

This operational equivalence reveals that what we traditionally call 'the complex plane' in analytic number theory is not merely a static arena but is, in fact, isomorphic to a 2D phase-space reconstruction of underlying computational dynamics. Consequently, the behavior of functions like $(s)(s)$, when studied through finite computation, is inherently governed by the geometric and topological constraints of dynamical attractors, bridging a fundamental gap between analysis and dynamical systems theory.

Theorem: For appropriate delay τ_{opt}, Takens embedding $[x(t), x(t-\tau_{\mathrm{opt}})]$ is equivalent to $[x(t), \mathcal{H}x(t)]$. Hence, complex plane and phase-space embedding describe the same geometry.

The Zeta Function as Dynamical System

Prime Distribution as Structured Signal

The prime counting function $\pi(x) \sim x/\ln x$ exhibits deviations structured by oscillations connected to $\zeta(s)$ zeros.

Definition

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \qquad s = \sigma + it.$$

Under GF, this is a symbolic computation with finite partial sums.

Euler product:

$$\zeta(s) = \prod_{p\,\text{prime}} \frac{1}{1-p^{-s}},$$

and functional equation

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s).$$

Zeros exhibit symmetry about $\sigma = 1/2$.

Alphonic Resolution and Symbolic Geometry

Before resolving the location of the critical line, we must develop a precise framework for understanding how numerical bases structure measurement space. This framework, which we call *alphonic resolution*, makes explicit the geometric properties of symbolic number systems that classical analysis typically leaves implicit.

Under Assumption PC, numerical bases are regarded as mere notational variants—different ways of writing the same abstract numbers. Under Assumption GF, however, bases are geometric substrates with measurable structural properties that affect computational behavior. The distinction between even and odd bases, in particular, creates geometric incommensurability that has profound implications for where attractors form in computational phase space.

Alphonic Number Representation

Under GF, every numerical representation consists of three components that reflect its character as a finite, measured symbolic construct.

Alphonic Representation A number in alphonic representation consists of:

1. **Alphon** ($\mathcal{A}$): the base—the finite set of available symbols per positional slot. Denoted $\mathcal{A}_n$ for base-n, where $|\mathcal{A}_n| = n$.

2. **Nixel** ($\mathcal{N}$): the integer part—a discrete position marker using alphon characters. Nixels are specific symbols from the alphon set, not abstract integers.

3. **Fracton** ($\mathcal{F}$): the fractional part—refinement beyond the alphon point, specifying sub-nixel resolution through additional alphon characters.

Example 5.1 (Base-10).

$$\mathcal{A}_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

(10 symbols). $N = 3$, $F = 0.14159$ (sequence: 1, 4, 1, 5, 9).

Example 5.2 (Base-37).

$$\mathcal{A}_{37} = \{0, 1, \ldots, 9, A, B, \ldots, Z, \alpha\}$$

(37 symbols). $N = 3$, $F = 0.I4G2$ (sequence: I, 4, G, 2).

This is not mere terminology. It reveals geometric structure:

- Nixels occupy discrete positions (countable, separated)
- Fractons occupy refinement space (dense, continuous within measurement limits)
- The alphon determines both: symbol count constrains geometry

Key insight: Under GF, these are not representations of Platonic objects but measurement specifications. Each nixel–fracton pair defines a fuzzy region in number space, not a perfect point.

Nixels: Discrete Geometric Markers

Nixels are not abstract integers from $\mathbb{R}$ (as under PC). They are *characters*—specific symbols with measurable properties.

Properties of nixels:

- Finite geometric form: each nixel has measurable shape when written or displayed.
- Discrete positions: nixels occupy separated locations in the alphon sequence.

- No intermediate states: one cannot be "between" nixel 3 and nixel 4.
- Countable: the set of nixels in any alphon is finite and enumerable.

Base-10 nixels:

$$\mathcal{N}_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

Ten discrete character positions spanning 9 units (the span differs from the count). Base-37 nixels:

$$\mathcal{N}_{37} = \{0, 1, 2, \ldots, 9, A, B, C, \ldots, Z, \alpha\},$$

spanning 36 units.

Geometric consequence: The parity of $|\mathcal{A}_n|$ (even vs. odd) determines whether the geometric centre falls *between* nixels (even) or *on* a nixel (odd).

Example 5.3 (Base-10 centre). Ten nixels: 0–9. Midpoint: $(0 + 9)/2 = 4.5 \Rightarrow$ between nixel 4 and 5 (fracton space).

Example 5.4 (Base-37 centre). Thirty-seven nixels: 0–α. Midpoint: $(0 + 36)/2 = 18 \Rightarrow$ on nixel I (nixel space).

Fractons: Fractal Uncertainty Refinement

The fracton space represents refinement beyond the alphon point—the region where we specify sub-nixel resolution. Each additional fracton character provides finer localization within measurement space.

Fractal Uncertainty Reduction For a measurement specified to k fracton digits in base-n, the uncertainty bounds are:

$$\delta_k = \frac{1}{2n^k},$$

where δ_k is the radius of the uncertainty region.

Proof sketch. Each fracton position divides the previous uncertainty interval into n subintervals. After k divisions, the interval width is $1/n^k$, giving radius $\delta_k = 1/(2n^k)$. □

Example 5.5 (Base-10). Each additional digit reduces uncertainty by factor 10.

Specification	Uncertainty δ	Reduction Factor
$\pi \approx 3$	± 0.5	—
$\pi \approx 3.1$	± 0.05	$\times 10$
$\pi \approx 3.14$	± 0.005	$\times 100$
$\pi \approx 3.141$	± 0.0005	$\times 1000$

Example 5.6 (Base-37). Higher alphon $\Rightarrow$ faster refinement: $\delta_k = \frac{1}{2 \cdot 37^k}$.

This refinement is fractal: self-similar at each level, scaling by factor n.

Geometric Incommensurability: Even vs. Odd Alphons

Alphon Parity An alphon $\mathcal{A}_n$ is *even* if $n \equiv 0 \pmod 2$, and *odd* if $n \equiv 1 \pmod 2$.

Geometric Dichotomy For $\mathcal{A}_n$ with positions $\{0, 1, \ldots, n-1\}$:

- If n is even, the geometric centre lies between two nixels (fracton space).
- If n is odd, the geometric centre lies on a nixel (nixel space).

Proof. centre $c = (0 + (n-1))/2 = (n-1)/2$. For even $n = 2m$: $c = m - 0.5$ (between integers). For odd $n = 2m+1$: $c = m$ (integer position). □

Examples. Base-10 ($n = 10$): $c = 4.5 \Rightarrow$ fracton centre. Base-37 ($n = 37$): $c = 18 \Rightarrow$ nixel centre.

Geometric Incommensurability Two alphons $\mathcal{A}_m$ and $\mathcal{A}_n$ are geometrically incommensurable if:

1. One has even parity and the other odd, or
2. $\gcd(m, n) = 1$ (coprime sizes).

Example 5.9. Base-10 vs. base-37: $\gcd(10, 37) = 1 \Rightarrow$ no alignment of discrete marks.

Information Density and Measurement Resolution

Information Capacity An alphon $\mathcal{A}_n$ with n symbols encodes:

$$I_{\text{bits}} = \log_2(n)$$

bits per character.

Alphon	n	Bits/char	Resolution
Binary	2	1.00	Coarse
Decimal	10	3.32	Medium
Hexadecimal	16	4.00	Fine
Base-37	37	5.21	Very fine

Base-37 thus encodes $\sim 57\%$ more information per character than base-10, achieving finer measurement resolution.

Measurement as Fuzzy Spheres

All measurements specify finite regions, not points. In d dimensions, uncertainty scales as $\delta_k = 1/(2n^k)$, giving:

$$V_k = \frac{4}{3}\pi\delta_k^3 = \frac{\pi}{6n^{3k}}.$$

Each fracton digit shrinks the fuzzy sphere volume by factor n^3.

Example 5.11. Base-10 measurement refinement produces $V_k \propto 10^{-3k}$—progressively smaller but never zero.

The Alphon Point and Geometric Transition

Alphon Point The boundary between nixel and fracton space, where discrete enumeration ends and fractal refinement begins.

Geometric interpretation:

- Before: discrete jumps between nixels.
- At: transition threshold.
- Beyond: fractal refinement through fractons.

Example 5.12. In 3.14159, nixel space ends at 3, alphon point at the dot, fracton space begins with 1,4,1,5,9.

Even bases $\Rightarrow$ centre in fracton space. Odd bases $\Rightarrow$ centre in nixel space.

Critical Line as Alphonic centre

When computing $\zeta(s)$ with $s = \sigma + it$, the real part σ is represented in some alphon.

Base-10 (even). centre: halfway between nixel 0 and 1 $\Rightarrow$ fracton position 0.5 (continuous-like). Normalized midpoint: $4.5/9 = 0.5$.

Base-37 (odd). centre: on nixel I (discrete). Normalized midpoint: $18/36 = 0.5$. Same numerical value, different geometric type.

Analogy: In a 10-gon, "halfway" lies on an edge; in a 37-gon, on a vertex. Both are "0.5" numerically, but geometrically distinct.

Implications for Computational Dynamics

Substrate-Dependent Attractors In a symmetric dynamical system computed within alphon $\mathcal{A}_n$, attractors form at the geometric centre of $\mathcal{A}_n$. The geometric type of this centre (fracton vs. nixel) depends on whether n is even or odd.

For the Riemann zeta function:

- Base-10 (even): attractor at fracton position 0.5.
- Base-37 (odd): attractor at nixel position I.

This difference is geometric necessity under finite symbolic computation.

Summary: Alphonic Resolution Framework

We have established:

- Alphonic representation: $(\mathcal{A}, \mathcal{N}, \mathcal{F})$ structure.
- Even/odd dichotomy: fracton-space vs. nixel-space centres.
- Fractal refinement: uncertainty $\delta_k = 1/(2n^k)$.
- Incommensurability: mismatched geometric lattices cannot align.
- Information density: $I_{\text{bits}} = \log_2 n$.
- Measurement as fuzzy spheres: finite uncertainty, never zero.
- Attractor dependence: determined by alphonic geometric centre.

Thus, the classical question

$$\text{“Why do zeros lie at } \Re(s) = 0.5?''$$

becomes, under GF:

“Why do zeros cluster at the alphonic geometric centre of the comput

Answer: geometric necessity—attractors form at symmetry points, and the symmetry point of base-10 is the fracton position 0.5.

The Geometric centre: Geofinitist Resolution

Base-10 Arithmetic Structure

Having established the alphonic framework in Section 5, we can now precisely identify the geometric centre of base-10 computational space and demonstrate why zeros of $\zeta(s)$ must cluster there.

Symbols $\{0, 1, \ldots, 9\}$ span $[0, 9]$, midpoint $4.5/9 = 0.5$. This is the geometric centre of the computational symbol set.

Attractor Symmetry

In symmetric dynamical systems, attractors form at geometric centres. Thus, $\text{Re}(s) = 0.5$ corresponds to the phase-space equilibrium of base-10 computation.

In applied mathematics, we recognize that all computations are dynamical processes with an inherent geometric structure defined by their symbolic substrate. The observed clustering of $(s)(s)$ zeros is therefore not merely accumulating numerical evidence for a Platonic truth, but is the natural attractor behaviour of the prime-distribution dynamics when computed in base-10 symbolic space. The symmetry of the functional equation finds its dynamical expression in this convergence toward the geometric centre of the computational phase space.

Theorem (Geofinitist Resolution of RH): Under GF, zeros of $\zeta(s)$ cluster at $\text{Re}(s) \approx 0.5$ within measurement precision because this is the geometric centre of base-10 space.

Implications and Extensions

Base-Dependence of the Critical Line

As demonstrated in Section 5, even and odd alphons have geometrically incommensurable structures. Base-10's fracton centre at 0.5 cannot be directly compared to base-37's nixel centre at 'I' without acknowledging they are different geometric types. If $\zeta(s)$ were computed in base-n arithmetic, zeros would cluster at the geometric centre of that base. For even bases, this remains 0.5; for odd bases, it aligns with a discrete symbol position.

Why This Connection Was Missed

Platonic assumptions, disciplinary silos, and neglect of measurement foundations obscured the computational geometric basis of RH.

Relationship to Other Approaches

GF complements classical, statistical, and computational methods by providing geometric explanation under finite conditions.

Conclusion

Under GF, the Riemann Hypothesis is resolved through alphonic geometry: zeros cluster at Re(s) = 0.5 in base-10 because this is the fracton-space centre of an even alphon with 10 discrete nixels. The alphonic framework reveals that computation in different bases produces attractors at different geometric locations—fracton centres for even alphons, nixel centres for odd alphons—making the critical line measurement-system-dependent. Mathematics is measurement; symbols are finite; alphonic resolution determines geometric structure; attractors form at computational symmetry points.

Summary

The clustering of zeros at $\mathrm{Re}(s) = 1/2$ arises from:

- Equivalence of complex and phase-space embeddings.
- Base-10 midpoint geometry at 0.5.
- Attractor dynamics inherent to symmetric systems.

Under GF, this is a structural consequence of computation, not a Platonic mystery.

Implications for Mathematics

1. Mathematics is finite measurement, not infinite abstraction.
2. Base structure influences results.
3. Proofs require provenance and measurement context.
4. Attractor geometry unites number theory and dynamics.

A Dynamical View of Applied Mathematics

This Geofinitist Resolution demonstrates that many problems in pure mathematics, when viewed through the lens of applied computation, transform into questions about dynamical system behavior rather than infinite, axiomatic claims. The tools of dynamical systems theory—attractors, phase space reconstruction, and stability analysis—provide a natural and powerful framework for understanding the patterns that robustly emerge in finite mathematical practice. This shifts the epistemological goal from proving eternal, universal truths to explaining stable, observable phenomena within a given measurement and computational framework.

Open Questions

- Base-37 computation of $\zeta(s)$ to test predictions.
- Generalization to other L-functions.
- Formalization of Geofinitist proof framework.

Final Remarks

Under GF, the Riemann Hypothesis is resolved as a *geometric attractor phenomenon.* Zeros cluster at $\mathrm{Re}(s) = 1/2$ because 0.5 is the geometric centre of base-10 symbolic space. Mathematics is measurement; symbols are finite; geometry governs computation.

Acknowledgments

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Takens' Theorem (Formal Statement)

Let M be a compact manifold of dimension d. For generic pairs (ϕ, h) with $\phi : M \to M$ a diffeomorphism and $h : M \to \mathbb{R}$ a measurement function, the delay map

$$\Phi_{(\phi,h)}(x) = (h(x), h(\phi(x)), \ldots, h(\phi^{2d}(x)))$$

is an embedding into $\mathbb{R}^{2d+1}$.

Hilbert Transform Properties

$$\mathcal{H}f(t) = \frac{1}{\pi}\text{P.V.}\int_{-\infty}^{\infty} \frac{f(\tau)}{t-\tau}\, d\tau.$$

Key properties: linearity, 90° phase shift, involution $\mathcal{H}[\mathcal{H}[f]] = -f$.

Base-10 Geometric centre

Symbol set $\{0, \ldots, 9\}$ spans $[0, 9]$. Midpoint: $(0+9)/2 = 4.5$, normalized $4.5/9 = 0.5$.

Base-37 Analysis

Symbols $\{0 - 9, A - Z, \alpha\}$ (37 total). centre position: 18, normalized $18/36 = 0.5$. Geometric character differs—centre lies on a symbol, not between symbols.

PC vs. GF Comparison

Aspect	PC (Platonic Continuum)	GF (Geofinitist Finite)
Numbers	Infinite, complete fields	Finite symbolic constructs
Computation	Idealized, exact	Finite precision, bounded uncertainty
Infinity	Completed object	Procedural instruction
Equality	Exact identity	Within measurement bounds
Proof	Logical deduction	Geometric resolution with provenance
Base	Irrelevant	Geometric substrate
Verification	Infinite, untestable	Finite, testable predictions
Epistemology	Eternal truth	Finite measurement realism