The Attralucian Essays:

Exploring the Finite

First Edition

The Attralucian Essays

On the Charge-Mass, Interaction Identity and Fine Structure Constant: A Geofinite Reading

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Chapter 1

First Reading: On the Charge-Mass Interaction Identity and Fine Structure Constant

Abstract for the First Reading

The fine-structure constant α is conventionally treated as a dimensionless constant – a pure number that characterises the strength of the electromagnetic interaction. This reading re-examines α through the lens of Geofinitism and Finite Symbolic Mechanics (FSM). Beginning from standard classical relations, an algebraic rearrangement is performed that places the electron charge–mass identity on one side and a combination of Rydberg recurrence, electromagnetic response, and α^3 on the other. The resulting expression

$$\frac{m_e}{e^2} \sim \frac{f_{\text{Rydberg}}\,\mu_0^{3/2}\,\varepsilon_0^{1/2}}{\alpha^3}$$

is not presented as a new calculation of α. Rather, it is read as a finite symbolic compression: the fine-structure constant appears as a cubic term mediating between charge–mass identity, atomic recurrence, and vacuum response. The classical reading treats this as a consistency relation among separate constants. The Geofinite reading treats it as a symbolic trace of a single, finite, measurement-based interaction.

Overview

The fine-structure constant, usually denoted by α, is one of the most compact and symbolically powerful quantities in modern physics. In the classical framing, it is treated as a dimensionless constant governing the strength of electromagnetic interaction. It is often written as a pure number, approximately

$$\alpha \approx 7.297 \times 10^{-3} \approx \frac{1}{137}. \tag{1.1}$$

Within conventional physics, the dimensionless nature of α gives it a special status. Because it does not depend on ordinary unit choices in the same way as dimensional quantities, it is often treated as a particularly deep numerical feature of the physical world.

However, from the standpoint of Finite Symbolic Mechanics (FSM), this interpretation is incomplete. A dimensionless number is not therefore a commitmentless object. It is still a symbolic compression. It is still written in a base. It still depends on a chain of measurements, definitions, substitutions, and representational decisions. It still arises from a finite symbolic and experimental process.

The purpose of this essay is not to claim that algebra alone establishes a new physics. Rather, the purpose is to show that ordinary algebra, applied to standard classical relations, exposes a form that is naturally interpretable within the Geofinite and FSM framing. In particular, the rearranged relation places the electron charge–mass identity on one side and Rydberg-scale recurrence, electromagnetic response, and fine-structure compression on the other.

The final relation reached is

$$\frac{m_e}{e^2} \sim \frac{f_{\mathrm{Rydberg}} \mu_0^{3/2} \varepsilon_0^{1/2}}{\alpha^3}. \tag{1.2}$$

This expression should not be read merely as a new way to calculate α. Rather, it shows where α sits within a finite symbolic relation involving electron charge–mass identity, Rydberg recurrence, and electromagnetic response.

The trajectory matters. The result is not obtained by inventing a symbolic form and then forcing physics into it. It begins with inherited classical relations, follows admissible algebraic transformations, and then asks what the resulting expression says when read through the commitments of Geofinitism and FSM.

Historical and Conceptual Provenance

The fine-structure constant entered physics through the study of atomic spectra. It is associated with the fine splitting of spectral lines and became central in the development of relativistic atomic theory and later quantum electrodynamics. In conventional form, it is often written as

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}, \tag{1.3}$$

where e is the elementary charge, ε_0 is the vacuum permittivity, $\hbar$ is the reduced Planck constant, and c is the speed of light.

Using $h = 2\pi\hbar$, this may also be written as

$$\alpha = \frac{e^2}{2h\varepsilon_0 c}. \tag{1.4}$$

Since

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}, \tag{1.5}$$

we may write

$$\frac{1}{\varepsilon_0 c} = \frac{1}{\varepsilon_0}\sqrt{\mu_0\varepsilon_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{1.6}$$

Therefore,

$$\alpha = \frac{e^2}{2h}\sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{1.7}$$

This is the starting point for the present derivation.

In classical physics, this expression is taken as a relation among separately defined constants. In the Geofinite reading, the expression is treated instead as a symbolic compression of a finite measurement structure. The question changes from:

> What is the numerical value of α?

to:

> What finite symbolic and measurement relations are compressed into the object conventionally called α?

This shift is central. The goal is not numerology around 1/137. The goal is to understand how the fine-structure constant participates in a measured relational structure involving charge, mass, recurrence, and electromagnetic response.

Rydberg Recurrence and the Re-expression of h

The earlier Finite Mechanics derivation begins by using the Rydberg frequency as an observational foothold. The Rydberg frequency is defined from the Rydberg constant R_∞ and the locally measured speed of light:

$$f_{\text{Rydberg}} = R_\infty c. \tag{1.8}$$

In the derivation used here, the Rydberg frequency is related to electron mass, charge, Planck's constant, and vacuum permittivity by

$$f_{\text{Rydberg}} = \frac{m_e e^4}{8h^3\varepsilon_0^2}. \tag{1.9}$$

This expression can be solved for h. First multiply both sides by $8h^3\varepsilon_0^2$:

$$8f_{\text{Rydberg}}h^3\varepsilon_0^2 = m_e e^4. \tag{1.10}$$

Now divide by $8f_{\text{Rydberg}}\varepsilon_0^2$:

$$h^3 = \frac{m_e e^4}{8f_{\text{Rydberg}}\varepsilon_0^2}. \tag{1.11}$$

Taking the cube root gives

$$h = \left(\frac{m_e e^4}{8 f_{\mathrm{Rydberg}} \varepsilon_0^2} \right)^{1/3}. \tag{1.12}$$

This step is already conceptually important. Planck's constant is not being treated here as an isolated primitive explanatory object. It is being expressed through measurable quantities associated with charge, electron mass, vacuum response, and Rydberg recurrence.

In the classical basin, this is an algebraic substitution. In the FSM basin, it is a symbolic decompression: a familiar constant is opened into a finite measurement relation.

Substitution into the Fine-Structure Constant

We now return to the fine-structure expression

$$\alpha = \frac{e^2}{2h} \sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{1.13}$$

Substituting

$$h = \left(\frac{m_e e^4}{8 f_{\mathrm{Rydberg}} \varepsilon_0^2} \right)^{1/3} \tag{1.14}$$

gives

$$\alpha = \frac{e^2}{2} \left(\frac{m_e e^4}{8 f_{\mathrm{Rydberg}} \varepsilon_0^2} \right)^{-1/3} \sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{1.15}$$

Using

$$\left(\frac{A}{B}\right)^{-1/3} = \left(\frac{B}{A}\right)^{1/3}, \tag{1.16}$$

we obtain

$$\alpha = \frac{e^2}{2}\left(\frac{8f_{\text{Rydberg}}\varepsilon_0^2}{m_e e^4}\right)^{1/3}\sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{1.17}$$

This may be written as

$$\alpha = \frac{e^2\left(8f_{\text{Rydberg}}\varepsilon_0^2\right)^{1/3}}{2\left(m_e e^4\right)^{1/3}}\sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{1.18}$$

Since

$$\left(m_e e^4\right)^{1/3} = m_e^{1/3} e^{4/3}, \tag{1.19}$$

we have

$$\alpha = \frac{e^2\left(8f_{\text{Rydberg}}\varepsilon_0^2\right)^{1/3}}{2e^{4/3} m_e^{1/3}}\sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{1.20}$$

Collecting the powers of e,

$$\frac{e^2}{e^{4/3}} = e^{2-4/3} = e^{2/3}. \tag{1.21}$$

Thus,

$$\alpha = \frac{e^{2/3}\left(8 f_{\mathrm{Rydberg}} \varepsilon_0^2\right)^{1/3}}{2 m_e^{1/3}} \sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{1.22}$$

Now observe that

$$8^{1/3} = 2. \tag{1.23}$$

Therefore,

$$\left(8 f_{\mathrm{Rydberg}} \varepsilon_0^2\right)^{1/3} = 2\left(f_{\mathrm{Rydberg}} \varepsilon_0^2\right)^{1/3}. \tag{1.24}$$

The factor of 2 produced by the cube root cancels with the denominator 2, giving

$$\alpha = \frac{e^{2/3}\left(f_{\mathrm{Rydberg}} \varepsilon_0^2\right)^{1/3}}{m_e^{1/3}} \sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{1.25}$$

This can be written more compactly as

$$\alpha = \left(\frac{e^2 f_{\mathrm{Rydberg}} \varepsilon_0^2}{m_e}\right)^{1/3} \sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{1.26}$$

This is the simplified Rydberg-based expression for the fine-structure constant used in the present analysis.

The Cubic Form

The expression

$$\alpha = \left(\frac{e^2 f_{\text{Rydberg}} \varepsilon_0^2}{m_e}\right)^{1/3} \sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{1.27}$$

contains a cube root. To expose the underlying cubic relation, cube both sides:

$$\alpha^3 = \left[\left(\frac{e^2 f_{\text{Rydberg}} \varepsilon_0^2}{m_e}\right)^{1/3} \sqrt{\frac{\mu_0}{\varepsilon_0}}\right]^3. \tag{1.28}$$

Using

$$\left(A^{1/3}\right)^3 = A \tag{1.29}$$

and

$$\left(\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)^3 = \left(\frac{\mu_0}{\varepsilon_0}\right)^{3/2}, \tag{1.30}$$

we obtain

$$\alpha^3 = \frac{e^2 f_{\text{Rydberg}} \varepsilon_0^2}{m_e} \left(\frac{\mu_0}{\varepsilon_0}\right)^{3/2}. \tag{1.31}$$

Now simplify the permittivity and permeability terms:

$$\varepsilon_0^2 \left(\frac{\mu_0}{\varepsilon_0}\right)^{3/2} = \varepsilon_0^2 \frac{\mu_0^{3/2}}{\varepsilon_0^{3/2}}. \tag{1.32}$$

Since

$$\varepsilon_0^2 = \varepsilon_0^{4/2}, \tag{1.33}$$

we have

$$\frac{\varepsilon_0^{4/2}}{\varepsilon_0^{3/2}} = \varepsilon_0^{1/2}. \tag{1.34}$$

Therefore,

$$\varepsilon_0^2 \left(\frac{\mu_0}{\varepsilon_0}\right)^{3/2} = \mu_0^{3/2}\varepsilon_0^{1/2}. \tag{1.35}$$

The cubic form becomes

$$\alpha^3 = \frac{e^2 f_{\text{Rydberg}}}{m_e}\mu_0^{3/2}\varepsilon_0^{1/2}. \tag{1.36}$$

Multiplying both sides by m_e, we obtain

$$m_e\alpha^3 = e^2 f_{\text{Rydberg}}\mu_0^{3/2}\varepsilon_0^{1/2}. \tag{1.37}$$

This is a central relation for the present essay.

Placing the Charge–Mass Relation on the Left

The preceding expression may be rearranged by dividing both sides by e^2:

$$\frac{m_e}{e^2}\alpha^3 = f_{\text{Rydberg}}\mu_0^{3/2}\varepsilon_0^{1/2}. \tag{1.38}$$

Dividing both sides by α^3, we obtain

$$\frac{m_e}{e^2} = \frac{f_{\text{Rydberg}}\mu_0^{3/2}\varepsilon_0^{1/2}}{\alpha^3}. \tag{1.39}$$

In FSM notation, strict equality is replaced by the tilde relation:

$$\frac{m_e}{e^2} \sim \frac{f_{\text{Rydberg}}\mu_0^{3/2}\varepsilon_0^{1/2}}{\alpha^3}. \tag{1.40}$$

The use of $\sim$ is not decorative. It records a change in interpretation. The equation is not being presented as a Platonic equality between abstract objects. Rather, it is being read as a finite symbolic correspondence between measured quantities, symbolic projections, and representational commitments.

This is the point at which the expression becomes especially interesting for Geofinitism.

Classical Reading

In the classical framing, the expression

$$\frac{m_e}{e^2} = \frac{f_{\text{Rydberg}}\mu_0^{3/2}\varepsilon_0^{1/2}}{\alpha^3} \tag{1.41}$$

would not normally be treated as revealing a charge–mass identity. It would be treated as an algebraic rearrangement within the accepted network of physical constants.

The electron mass m_e, the elementary charge e, the Rydberg frequency f_{Rydberg}, the vacuum permeability μ_0, the vacuum permittivity ε_0, and the fine-structure constant α are taken as separately defined quantities connected through established physical equations.

From this point of view, the result is a consistency relation. It shows that the constants are mutually constrained, but it does not challenge the assumption that mass and charge are separable properties.

This is a valid reading within the classical basin. The algebra is ordinary. The transformations are standard. Nothing in the derivation alone compels a new physical interpretation.

However, the same algebra can be read differently when the foundational commitments change.

FSM Reading

In FSM, charge and mass are not treated as separate primitive things. They are treated as projections of a combined interactional identity.

Thus, the term

$$\frac{m_e}{e^2} \tag{1.42}$$

is not read merely as “mass divided by charge squared.” Rather, it is interpreted as a finite symbolic projection of the electron charge–mass interactional identity.

The electron is not first a mass to which charge is externally attached. Nor is it first a charge that separately carries mass. Instead, m_e and e are symbolic projections of a single measured interactional identity.

The right-hand side then becomes highly suggestive:

$$\frac{f_{\text{Rydberg}}\mu_0^{3/2}\varepsilon_0^{1/2}}{\alpha^3}. \tag{1.43}$$

Here f_{Rydberg} introduces recurrence. It points toward the measurable atomic-scale cadence associated with hydrogenic spectral structure.

The factors $\mu_0^{3/2}$ and $\varepsilon_0^{1/2}$ introduce electromagnetic response. They represent the local electromagnetic terms through which the relation is expressed.

The denominator α^3 is especially important. In this reading, the fine-structure constant is not the thing being solved for. Instead, it appears as a cubic compression term mediating the relationship between electron charge–mass identity, Rydberg recurrence, and electromagnetic response.

The expression may therefore be read as follows:

> The electron charge–mass identity corresponds to Rydberg-scale recurrence and electromagnetic response, mediated by the cubic compression of the fine-structure constant.

Or, in a more compact form:

> The fine-structure constant is not merely a dimensionless scalar. It is a symbolic compression term within a finite interactional relation.

Why This Is Not Merely Arbitrary Algebra

A natural objection is that the derivation is only algebra. In a narrow technical sense, this is true. The transformation follows from standard algebraic rearrangement of accepted relations.

However, not all algebraic rearrangements are conceptually equal. Many rearrangements are inert: they move symbols without opening any new interpretive structure. This rearrangement does something more interesting. It places the electron charge–mass relation on one side and the Rydberg–electromagnetic–fine-structure relation on the other.

That arrangement is not random from the standpoint of

FSM. It aligns with several prior commitments:

- charge and mass are not independent primitive substances;
- physical constants are symbolic stabilisations of measurement relations;
- Rydberg-scale recurrence is an important observational foothold;
- electromagnetic response terms should not be hidden behind c alone;
- base and symbolic representation are not invariant in FSM;
- finite symbolic expressions require provenance and uncertainty.

The result should therefore not be overstated as a proof of FSM. It is better understood as a compatibility result. A standard classical relationship can be rearranged into a form that is unusually well aligned with the FSM interpretation of charge, mass, recurrence, and electromagnetic response.

The significance is not that algebra alone produces a new physical law. The significance is that ordinary algebra, applied to relations already validated within classical physics, exposes a form that can be naturally interpreted through Geofinite commitments.

The FSM reading is not externally imposed. It is available within the inherited equations.

Narrative Reading of the Final Relation

The final relation is

$$\frac{m_e}{e^2} \sim \frac{f_{\mathrm{Rydberg}}\mu_0^{3/2}\varepsilon_0^{1/2}}{\alpha^3}. \tag{1.44}$$

Read as a symbolic sentence, this says:

> The electron's charge–mass identity is structurally related to Rydberg-scale recurrence, local electromagnetic response, and fine-structure compression.

The left-hand side presents the electron as a charge–mass relation. It is not a small object with a separable mass and charge attached to it. It is a measured interactional identity whose symbolic projections include mass-like and charge-like behaviour.

The right-hand side then tells us what this identity is embedded within. The Rydberg frequency introduces recurrence. The electromagnetic response terms introduce the local response structure. The fine-structure constant appears as a cubic compression term.

Thus, the equation may be read as saying:

> The measured charge–mass identity of the elec-

> tron corresponds to an atomic recurrence scale, modulated by electromagnetic response, and stabilised through the cubic compression represented by the fine-structure constant.

This is not a replacement for the classical reading. It is a different reading of the same symbolic structure under different foundational commitments.

Base Non-Invariance and Symbolic Compression

The fine-structure constant is conventionally treated as dimensionless and therefore independent of the system of units. In FSM, this does not make it representation-independent.

The decimal expression

$$\alpha \approx 0.00729735256 \tag{1.45}$$

and the reciprocal expression

$$\alpha^{-1} \approx 137.036 \tag{1.46}$$

are not the same symbolic object, even if they refer to the same conventional numerical relation. A binary expansion, hexadecimal expansion, decimal truncation, reciprocal decimal expression, or symbolic algebraic form each has a different finite representational structure.

In standard physics, these differences are usually treated as display choices. In FSM, they matter. A symbolic expression has a finite instantiation. It has length, truncation behaviour, error propagation, compression cost, and provenance.

Thus, the fine-structure constant is not merely a number independent of units. It is a finite symbolic compression whose representation depends on the alphonic system in which it is instantiated.

This does not mean the physical relation changes arbitrarily when the base changes. Rather, it means that the symbolic path through which the relation is represented changes. FSM therefore distinguishes between a conventional numerical correspondence and the finite symbolic geometry required to instantiate that correspondence.

Alphonic Containment and Finite Interactional Volume

The preceding relation remains algebraic unless a geometric interpretation is added. In FSM, symbolic representations are not treated as abstract objects detached from instantiation. Every symbolic representation requires finite conditions of representation.

At the alphonic limit, a symbolic representation has a minimum geometric containment. The most natural minimum containment is spherical. However, this should not be read as claiming that the electron is literally a sphere, nor that the symbolic relation has a physical body. That framing is too strong and too easily misunderstood.

A more careful formulation is to say that the charge–mass relation requires a finite interactional volume under uncertainty.

Let

$$\mathbb{V}^{\epsilon}_{\min}(\cdot) \tag{1.47}$$

denote the minimum finite interactional volume required to instantiate a symbolic relation within uncertainty ϵ.

Since charge and mass are not treated as separable primitive substances, the left-hand side may be written not merely as a fraction but as an interactional pairing:

$$m_e : e^2. \tag{1.48}$$

Here the colon denotes a combined charge–mass identity rather than a division of independent objects. The algebraic fraction remains useful for derivation, but the geometric reading treats the relation as a finite interactional identity requiring containment.

We may therefore write

$$\mathbb{V}^{\epsilon}_{\min}\left(m_e : e^2\right) \;\sim\; \mathbb{V}^{\epsilon}_{\min}\left(f_{\mathrm{Rydberg}}, \mu_0^{3/2}, \varepsilon_0^{1/2}, \alpha^{-3}\right). \tag{1.49}$$

This expression should not be read as replacing the algebraic relation. Rather, it gives the algebraic relation an FSM geometric interpretation.

It says that the minimum finite interactional volume of the electron charge–mass identity corresponds to the minimum finite interactional volume of the Rydberg–electromagnetic–fine-structure relation, within uncertainty.

This formulation preserves the algebra while adding the FSM requirement that symbolic relations must have finite representational conditions. The relation is therefore not merely numerical. It is geofinite: a finite interactional volume under uncertainty.

Discussion

The derivation developed here has two distinct readings.

In the classical reading, the expression

$$\frac{m_e}{e^2} = \frac{f_{\mathrm{Rydberg}}\mu_0^{3/2}\varepsilon_0^{1/2}}{\alpha^3} \tag{1.50}$$

is a rearrangement within the accepted constants network. It is an identity produced by substituting known relations and simplifying them. It does not by itself force a reinterpretation of charge, mass, or the fine-structure constant.

In the FSM reading, the same expression takes on a different role. The left-hand side is read as a projection of charge–mass identity. The right-hand side is read as recurrence, electromagnetic response, and fine-structure compression. The expression becomes a symbolic trace of a finite interactional relation.

This difference illustrates the broader Geofinite method. Geofinitism does not begin by rejecting inherited equations. It asks what those equations become when read through different foundational commitments.

The classical basin separates quantities first, then relates them through constants. The Geofinite basin begins from measured relations and treats separated quantities as symbolic projections of finite interaction processes.

The fine-structure constant is therefore an especially useful object of study. It sits at the intersection of atomic recurrence, electromagnetic interaction, charge, mass, measurement, and symbolic compression. It appears dimensionless, but its representation is not free from symbolic structure. It appears simple, but it compresses a complex provenance of theory, measurement, and metrology.

The cubic form is particularly suggestive. The appearance of α^3 shows that the relation is not merely linear. In this rearranged form, the fine-structure constant acts as a cubic compression term. This does not imply that the cube has a final physical interpretation at this stage, but it gives a structured direction for further investigation.

Likewise, the emergence of

$$\mu_0^{3/2}\varepsilon_0^{1/2} \tag{1.51}$$

should be treated cautiously. It would be premature to over-interpret the asymmetry between permeability and permittivity. However, within FSM it is legitimate to notice that the charge–mass relation, when placed in this form, is connected to a nontrivial electromagnetic response structure.

The most important conclusion is methodological. This derivation does not prove FSM, but it does show that FSM can enter classical physics through constrained symbolic pathways rather than arbitrary invention. The path follows accepted algebra. The result then becomes meaningful because it aligns with independently developed Geofinite commitments.

Conclusion

The fine-structure constant is often treated as one of physics' deepest dimensionless constants. In the Geofinite reading developed here, it is better understood as a finite symbolic compression within a wider measurement relation.

Starting from standard classical expressions, we derived

$$\alpha = \left(\frac{e^2 f_{\mathrm{Rydberg}} \varepsilon_0^2}{m_e} \right)^{1/3} \sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{1.52}$$

Cubing and rearranging gave

$$\frac{m_e}{e^2} \sim \frac{f_{\mathrm{Rydberg}} \mu_0^{3/2} \varepsilon_0^{1/2}}{\alpha^3}. \tag{1.53}$$

This final expression places the electron charge–mass relation on one side and the Rydberg–electromagnetic–fine-structure relation on the other. In the classical frame, it remains a consistency relation among constants. In the FSM frame, it becomes a finite symbolic trace of interactional identity.

The expression may therefore be read as saying:

> The electron charge–mass identity is structurally related to Rydberg-scale recurrence and electromagnetic response, mediated by the cubic compression of the fine-structure constant.

Finally, by introducing the minimum finite interactional volume under uncertainty,

$$\mathbb{V}^{\epsilon}_{\min}\left(m_e : e^2\right) \;\sim\; \mathbb{V}^{\epsilon}_{\min}\left(f_{\text{Rydberg}}, \mu_0^{3/2}, \varepsilon_0^{1/2}, \alpha^{-3}\right), \tag{1.54}$$

the algebraic relation is given a Geofinite geometric interpretation. It becomes not merely a ratio of quantities, but a finite interactional volume under uncertainty.

This is the central contribution of the present essay. The fine-structure constant is not treated as a mysterious isolated number, nor as a final object of numerological speculation. It is treated as a compression term within a finite symbolic relation. The inherited classical equations remain intact, but their meaning changes when read through the basin of Geofinitism.

In the classical basin, the constants relate.

In the Geofinite basin, the relation is prior, and the constants are stabilised symbolic projections.

Chapter 2

Second Reading: On the Missing Geometry of the Fine-Structure Relation

Abstract for the Second Reading

The algebraic relation derived in the First Reading is flat: it treats symbols as pure numbers or point-like properties, erasing their finite geometry. This Second Reading restores what classical notation omits: extent, volume, uncertainty, and provenance. Each symbol – m_e, e, μ_0, ε_0, f_{Rydberg}, and α – is reinterpreted as a Nexil: a finite symbolic unit generated at the Alphonic Limit. The exponents $3/2$, $1/2$, and the cubic α^3 are shown to be traces of missing geometry. The fine-structure constant is reread as a ratio of volumes – a cubic compression factor between the interactional volume of the charge–mass identity and the symbolic volume of its representation. The

asymmetric electromagnetic exponents (3/2 and 1/2) are interpreted as fractional signatures of how the measurement process projects onto different axes of the symbolic container. This reading does not predict a new value for α. It changes what α means: from a dimensionless constant to a metrological conversion factor with finite geometry.

From Algebraic Relation to Geometric Interpretation

The preceding chapter derived a cubic relation:

$$\frac{m_e}{e^2} \sim \frac{f_{\text{Rydberg}}\, \mu_0^{3/2}\, \varepsilon_0^{1/2}}{\alpha^3}$$

and offered a narrative reading: the electron charge–mass identity corresponds to Rydberg-scale recurrence and electromagnetic response, mediated by the cubic compression of the fine-structure constant.

But the equation, as written, is still *flat*. It treats m_e, e, μ_0, ε_0, f_{Rydberg}, and α as if they were pure numbers or point-like properties. This is the classical inheritance. The symbols have no extent, no volume, no uncertainty, no provenance. They are idealised.

Finite Mechanics and Finite Symbolic Mechanics together restore what classical notation erased: the *finite geome-*

try of the symbol itself.

2.1 Finite Mechanics and Finite Symbolic Mechanics

Finite Mechanics began with a simple observation: every measurement is finite. Every ruler has a smallest mark. Every sensor has a noise floor. Every clock ticks discretely. From this observation, a framework was built in which extent, uncertainty, and provenance are irreducible features of any measured quantity. The Alphonic Limit was introduced as the boundary of first-order measurement – the point at which a measurement becomes a finite symbolic distinction.

Finite Symbolic Mechanics extends this insight to the symbols we use to record measurements. The symbol is not a transparent window onto a measured quantity. It is itself a *finite geometric object* – what we call a *Nexil.* It is generated at the Alphonic Limit. It has volume. It has uncertainty. It carries the trace of its own generation (provenance). The classical habit of treating symbols as points or pure numbers is a flattening – a projection that discards the geometry of representation.

Thus, when we write an equation like

$$\frac{m_e}{e^2} = \frac{f_{\text{Rydberg}}\,\mu_0^{3/2}\,\varepsilon_0^{1/2}}{\alpha^3}$$

we are not relating four independent "things". We are relating *four symbolic compressions* of a single underlying measurement process. The equals sign, in the Geofinite frame, is replaced by the tilde:

$$\frac{m_e}{e^2} \sim \frac{f_{\mathrm{Rydberg}}\,\mu_0^{3/2}\,\varepsilon_0^{1/2}}{\alpha^3}$$

The tilde marks a correspondence under finite constraints, not a Platonic equality.

2.2 Extent as a Missing Parameter

What is missing from the classical reading of this equation is **extent**. The symbols m_e, e, f_{Rydberg}, μ_0, ε_0, and α are treated as points or pure numbers. But they are Nexils. They have finite geometry. Their geometric extent is not zero. It is larger than the Alphonic Limit – the smallest distinguishable symbolic volume.

Let $V_\alpha(X)$ denote the finite symbolic volume of Nexil X at the Alphonic Limit. Then each symbol in the equation carries such a volume. The equation, when restored to its full geometric meaning, is not a relation among pure numbers. It is a relation among volumes.

The appearance of exponents – 3/2, 1/2, and especially the cubic α^3 – is the trace of this missing geometry. The exponents tell us *how many dimensions* of the symbolic volume are being compressed, and in what proportion.

2.2.1 The Cubic Clue

The cube is the most important clue. A cube suggests three dimensions. Three independent measurement directions collapsed into one symbolic constant. The fine-structure constant, in this reading, is not "dimensionless". It is a *ratio of volumes*:

$$\alpha \sim \left(\frac{V_{\text{interaction}}}{V_{\text{symbolic}}}\right)^{1/3}$$

That is, the interactional volume of the electron charge–mass process, divided by the symbolic volume of the representation that stabilises it, raised to the one-third power. The cubic α^3 in the denominator of our relation is therefore the direct expression of this ratio:

$$\alpha^3 \sim \frac{V_{\text{interaction}}}{V_{\text{symbolic}}}$$

The fact that α^3 appears in the denominator means that the right-hand side of the equation is *inversely proportional* to the cubic compression. As the interactional volume grows relative to the symbolic volume, α increases – but the relation is cubic, so sensitivity is high.

2.2.2 The Asymmetric Electromagnetic Exponents

The terms $\mu_0^{3/2}$ and $\varepsilon_0^{1/2}$ are also traces of missing geometry. Their exponents are not integers. They are rational numbers: 1.5 and 0.5. This asymmetry is not accounted for in classical theory. It is usually treated as an artifact of the SI system – a convention.

But Geofinitism asks: what if it is not an artifact? What if the asymmetry is the *signature* of how the measurement process projects onto different axes of the symbolic container? The 3/2 exponent suggests that the permeability symbol μ_0 is compressed along three half-dimensions – a fractional scaling that reflects the geometry of the measurement apparatus. The 1/2 exponent suggests that ε_0 is compressed along one half-dimension. Together, they sum to 2 – the power of $1/c^2$ in the wave equation – but they are split unevenly.

A fully Geofinite equation would not flatten this asymmetry. It would preserve it as a geometric fact about the measurement process.

2.3 The Nexil and the Minimum Interactional Volume

We now introduce the *minimum finite interactional volume under uncertainty.* Let

$$\mathbb{V}^{\epsilon}_{\min}(X)$$

denote the smallest volume in which the symbolic relation X can be instantiated, given uncertainty ϵ. This is not a free parameter. It is bounded by the Alphonic Limit. It is the volume of the Nexil itself.

For the charge–mass identity, we may write:

$$\mathbb{V}^{\epsilon}_{\min}\left(m_e : e^2\right)$$

Here the colon denotes a combined interactional identity – not a division of independent objects, but a single process with two symbolic projections. The colon replaces the fraction bar when we move from algebraic compression to geometric interpretation.

Similarly, for the right-hand side:

$$\mathbb{V}^{\epsilon}_{\min}\left(f_{\text{Rydberg}}\right), \quad \mathbb{V}^{\epsilon}_{\min}\left(\mu_0^{3/2}\right), \quad \mathbb{V}^{\epsilon}_{\min}\left(\varepsilon_0^{1/2}\right), \quad \mathbb{V}^{\epsilon}_{\min}\left(\alpha^{-3}\right)$$

The full geometric correspondence is then: [1]

This is not a replacement for the algebraic relation. It

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[1] $$\mathbb{V}^{\epsilon}_{\min}\left(m_e : e^2\right) \sim \mathbb{V}^{\epsilon}_{\min}\left(f_{\text{Rydberg}}\right) \cdot \mathbb{V}^{\epsilon}_{\min}\left(\mu_0^{3/2}\right) \cdot \mathbb{V}^{\epsilon}_{\min}\left(\varepsilon_0^{1/2}\right) \cdot \mathbb{V}^{\epsilon}_{\min}\left(\alpha^{-3}\right)$$

is the algebraic relation *unflattened*. The algebra is preserved, but each symbol is now understood to carry its own finite geometric extent.

2.4 The Fine-Structure Constant as Compression Factor

From this geometric reading, the fine-structure constant is revealed as a *compression factor* between two regimes of measurement: the physical (charge, mass, recurrence) and the symbolic (the equation itself). It is not a dimensionless constant. It is a ratio of volumes – a metrological conversion factor.

The cubic form α^3 appears because the conversion is three-dimensional. The charge–mass identity occupies a volume in interaction space. The symbolic representation compresses that volume into a flat symbol. The compression is not linear. It is cubic because the symbol itself has three geometric dimensions (its extent in the Alphonic volume) and the interaction has three degrees of freedom (charge, mass, and their relation to recurrence and response).

Thus, the value $\alpha \approx 1/137$ is not a mystery. It is the measured ratio of two volumes, expressed in a particular symbolic base (decimal), under particular measurement conditions (our apparatus, our epoch, our Alphonic Limit).

In another epoch, with a different Alphonic Limit, α might be different – not because physics changes, but because the geometry of representation changes.

2.5 Why This Is Not Numerology

A cautious reader will object: this is just reinterpreting an algebraic relation, not deriving a new value for α. That is correct. The purpose is not to predict α. The purpose is to change what α *means*.

In the classical basin, α is a pure number – a dimensionless constant that appears to be a free parameter of the universe. In the Geofinite basin, α is a *measured compression factor* – a ratio of finite volumes, with uncertainty, provenance, and dependence on the symbolic system in which it is expressed.

This shift has consequences. If α is a ratio of volumes, then it is not universal in the Platonic sense. It is universal only within the epoch and apparatus that produced it. A different measurement apparatus – different resolutions, different base, different symbolic geometry – would produce a different α. Not because the interaction is different, but because the compression is different.

This is not relativism. It is *metrological accountability*. It restores what classical physics erased: the finite geometry of the symbol.

2.6 Conclusion of the Second Reading

The cubic relation derived in the first reading is not a new equation. It is the same equation, read with new eyes. The second reading adds what was missing: *extent.*

We have shown that:

- Each symbol in the fine-structure relation is a Nexil – a finite geometric object with volume, uncertainty, and provenance.
- The exponents (3/2, 1/2, and 3) are traces of missing geometry.
- The cube α^3 indicates that the fine-structure constant is a cubic compression factor – a ratio of interactional volume to symbolic volume.
- The asymmetric electromagnetic exponents (3/2 and 1/2) reflect the projection of the measurement process onto different axes of the symbolic container.
- The minimum finite interactional volume $\mathbb{V}^{\epsilon}_{\min}(\cdot)$ provides a geometric interpretation of the algebraic relation.

The fine-structure constant is not a dimensionless mystery. It is a *metrological conversion factor* between two regimes of measurement: the physical interactional volume of the electron and the symbolic volume of our representation. Its cubic form is the signature of a three-dimensional compression.

In the classical basin, constants relate.

In the Geofinite basin, the relation is prior, and constants are stabilised symbolic projections with finite geometry.

This is the contribution of Finite Symbolic Mechanics to the interpretation of the fine-structure constant. The algebra remains intact. The meaning changes. And the missing geometry is restored.

Chapter 3

Third Reading: On Spin, the Saddle Point, and the Inseparability of Charge and Mass

Abstract for the Third Reading

The First Reading revealed α as a cubic compression term; the Second Reading restored the missing geometry of extent and asymmetry. But both readings remain static. This Third Reading introduces dynamics. It asks: what happens when the finite, asymmetric, inseparable charge–mass interaction encounters an inhomogeneous magnetic field? The answer is the Stern–Gerlach experiment. The discrete splitting of the beam is not evidence of a new quantum degree of freedom called spin. It is the consequence of saddle-point geometry: a finite, asymmetric trajectory passing through a potential surface with exactly two exit valleys. Spin is reinterpreted

as a flattened narrative – a convenient compression of a complex geometric process. The cubic α^3 and the asymmetric exponents 3/2 and 1/2 are shown to be the static traces of this dynamic, chiral, saddle-point interaction. The reading closes the loop from algebra to geometry to experiment, demonstrating that the fine-structure constant and spin are not independent primitives but stabilised projections of a single finite measurement process.

From Static Compression to Dynamic Separation

The first reading of the fine-structure relation revealed α as a cubic compression term – a ratio of volumes, not a dimensionless constant. The second reading uncovered the asymmetric exponents 3/2 and 1/2 as traces of missing geometry and direction – the flattened remnants of chirality and the right-hand rule.

But these readings remain static. They treat the equation as a snapshot. The third reading adds what the equation cannot show: *dynamics*. It asks: what happens when the charge–mass interaction, with its finite asymmetry and irreducible chirality, encounters an inhomogeneous magnetic field?

The answer is the Stern–Gerlach experiment. And the key to understanding it is not spin – but the *saddle point*.

3.1 The Stern–Gerlach Experiment: A Classical Problem Re-Examined

In 1922, Otto Stern and Walther Gerlach passed a beam of silver atoms through an inhomogeneous magnetic field. Classical physics predicted that the continuous distribution of magnetic moments would produce a continuous smear on the detector. Instead, they observed two discrete spots – a split into two beams.

This result was interpreted as the first direct evidence of quantisation – of "spin" – a purely quantum degree of freedom with no classical analogue. Within the Geofinite framework, this interpretation is questioned. Not because the experiment was wrong, but because the classical model used to interpret it made inadmissible assumptions.

3.1.1 The Inadmissible Assumptions

The classical model assumed:

- **Separability**: that the magnetic moment of the atom is independent of its mass and charge distribution.
- **Isotropy**: that the interaction is symmetric with respect to orientation.
- **Infinite precision**: that the trajectory can be

treated as a continuous, deterministic line.

- **Platonic forms**: that "spin" exists as a property prior to measurement.

Geofinitism rejects each of these. Charge and mass are not separable. The interaction is not isotropic – it carries the chirality of the right-hand rule. Measurements are finite, with uncertainty and provenance. And properties are not prior to measurement; they are stabilised projections of measurement narratives.

3.2 The Saddle Point Geometry

When a charge–mass interaction – a single, inseparable, finite-asymmetric entity – passes through an inhomogeneous magnetic field, the field geometry is not uniform. In the Stern–Gerlach apparatus, the field has a *saddle point*: a region where the field gradient changes sign in different directions.

A saddle point is a geometric feature of a potential surface. It has one direction of negative curvature (unstable) and two directions of positive curvature (stable). In the Stern–Gerlach magnet, the saddle is oriented so that the charge–mass interaction, approaching along the axis of the beam, encounters a region where the force is zero (the saddle) but the gradient is non-zero on either side.

3.2.1 The Saddle as a Decision Point

Classical physics treats the saddle as a problem. Trajectories near a saddle are unstable – small perturbations lead to large divergences. In the classical model, this was seen as a difficulty: why would the beam split discretely rather than spreading continuously?

Geofinitism answers: because the charge–mass interaction is *not a point particle.* It has finite extent. It has internal asymmetry. When this finite, asymmetric object approaches the saddle, it cannot "choose" to go through the centre. The saddle forces a binary outcome: the interaction is deflected to one side or the other, depending on the orientation of its internal asymmetry relative to the field gradient.

The discreteness of the outcome is not due to quantisation. It is due to the *topology of the saddle.* A saddle has exactly two valleys. There is no continuum of stable exit trajectories – only two. The beam splits because the geometry of the potential forces it to split.

3.2.2 The Role of Finite Asymmetry

If the charge–mass interaction were perfectly symmetric (no internal chirality, no right-hand rule), the saddle would produce a continuous spread. The interaction would be equally likely to exit anywhere on the detector. But the interaction is *not* symmetric. The right-hand rule, the 3/2 and 1/2 exponents, the cubic α – all point to a fundamental asymmetry in the charge–mass identity.

This asymmetry, when coupled with the saddle geometry, produces exactly two stable exit trajectories. The two beams correspond to the two orientations of the asymmetry relative to the field gradient. In the classical quantum interpretation, these are called ”spin up” and ”spin down”. In the Geofinite interpretation, they are the two exit valleys of a saddle point, traversed by a finite, asymmetric charge–mass interaction.

3.3 The Historical Inseparability of Charge and Mass

The Stern–Gerlach experiment is often presented as evidence for a new property of the electron: spin. But spin is never measured directly. What is measured is the *separation of a beam* into two discrete spots. The interpretation of that separation as ”spin” depends on the prior assumption that charge and mass are separate properties, each with its own associated magnetic moment.

Geofinitism rejects that assumption. No experiment has ever measured the charge of an electron without also measuring its mass. No experiment has ever measured the mass of an electron without also measuring its charge. The two are not independent. They are projections – stabilised symbolic compressions – of a single interactional identity.

Thus, the Stern–Gerlach experiment does not demonstrate the existence of spin. It demonstrates that a finite, asymmetric, charge–mass interaction, when passed through a saddle-point magnetic field, splits into two trajectories. The split is discrete because the saddle has two exit valleys. The existence of two beams does not require a new quantum degree of freedom. It requires only the geometry of the saddle and the asymmetry of the interaction.

3.4 Spin as a Flattened Narrative

In the classical quantum narrative, spin is presented as a primitive – a ”built-in” angular momentum with no classical analogue. This narrative is appealing because it ”explains” the Stern–Gerlach result. But it does so by introducing a new entity, rather than by re-examining the assumptions of the classical model.

Geofinitism offers an alternative. The Stern–Gerlach result is explained by:

- The finite extent of the charge–mass interaction (it is not a point).
- The internal asymmetry of that interaction (the right-hand rule, chirality).

- The saddle-point geometry of the magnetic field.
- The inseparability of charge and mass (they are projections, not independent properties).

No new entity (spin) is required. The "spin" of the electron is a flattened narrative – a convenient fiction that compresses the complex, finite, asymmetric, saddle-point dynamics into a single symbolic label. Like the ket and the Heaviside function, spin is not wrong. It is *flattened*. It hides the finite geometry and the dynamic process that produced the measurement.

3.5 Connecting Back to the Fine-Structure Constant

The third reading now connects to the first two. The fine-structure relation:

$$\frac{m_e}{e^2} \sim \frac{f_{\text{Rydberg}}\,\mu_0^{3/2}\ \varepsilon_0^{1/2}}{\alpha^3}$$

is the static, flattened, scalar version of the dynamic, chiral, saddle-point interaction observed in the Stern–Gerlach experiment. The cubic α and the asymmetric exponents 3/2 and 1/2 are not arbitrary. They are the *trace* of the saddle geometry and the finite asymmetry.

- The 3/2 exponent corresponds to the three spatial dimensions of the saddle and the half-integer nature of the split (the "spin" 1/2). The saddle has three dimensions (two stable, one unstable); the half-integer emerges from the asymmetry of the charge–mass interaction, which has no classical analogue in the flattened spin narrative. - The 1/2 exponent corresponds to the binary outcome (two beams) compressed into a fractional power. - The cubic α^3 corresponds to the three-dimensional volume of interaction – the finite extent of the charge–mass identity as it passes through the saddle.

Thus, the fine-structure constant is not a coupling strength. It is a *compressed narrative of the Stern–Gerlach experiment* – and of every other experiment involving charge and mass. It is the static shadow of a dynamic, chiral, saddle-point interaction.

3.6 Conclusion of the Third Reading

The Stern–Gerlach experiment does not demonstrate the existence of spin. It demonstrates the behaviour of a finite, asymmetric, inseparable charge–mass interaction encountering a saddle-point magnetic field. The discrete separation is a consequence of the saddle topology, not of quantisation.

Spin is a flattened narrative – a convenient label that compresses a complex geometric and dynamic process.

The fine-structure constant, with its cubic form and asymmetric exponents, is the static signature of the same process. The 3/2 and 1/2 are traces of the saddle geometry and the binary outcome. The cubic α is the compression of the three-dimensional interaction volume.

In the classical basin, spin is a primitive, and α is a dimensionless constant. In the Geofinite basin, both are stabilised projections – finite symbolic compressions of a single, dynamic, chiral, saddle-point interaction. The history of measurement (charge and mass never separated) and the geometry of the apparatus (the saddle) together explain what classical physics needed a new quantum degree of freedom to explain.

This is the third reading. It closes the loop from static equation to dynamic experiment, from compression to separation, from flattened symbol to finite measurement.