The Attralucian Essays:

Exploring the Finite

First Edition

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FSM: The Foundations of Linear Mathematics

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Overview

Linear mathematics is often presented as the simplest and most self-evident branch of mathematical thought. The straight line, linear equations, slopes, vectors, and matrices are treated as foundational abstractions whose meaning appears timeless and universal. Yet the historical emergence of modern linear notation occurred remarkably late, and the operational reality of linearity has rarely been examined from the standpoint of finite measurement and symbolic construction.

This chapter develops a Geofinite and Finite Symbolic Mechanics (FSM) reinterpretation of the foundations of linear mathematics. Beginning from the concept of the straight line, we examine the historical development of linear representation from Euclidean geometry through Cartesian coordinates to the nineteenth-century stabilization of the slope-intercept form. We then reinterpret the straight line not as a Platonic object or infinite set of perfect points, but as a finite symbolic attractor generated through repeated nexil-to-nexil propagation within a bounded measurement manifold.

Within FSM, every symbolic representation emerges through finite measurement events. Symbols are finite measurable stabilizations rather than metaphysical absolutes. The equality sign is replaced by the Geofinite tilde operator ($\sim$), indicating bounded measurable correspondence rather than perfect identity. The straight line therefore becomes a persistent uncertainty-bounded geodesic within the finite symbolic manifold.

The chapter further explores how matrices, eigenvectors, coordinate systems, and aspects of quantum mechanics can be understood as higher-order compressions of finite symbolic propagation processes. Linear mathematics is reframed not as an abstract eternal system, but as a stabilized symbolic compression arising from finite operational procedures grounded in measurement.

Introduction: The Strange Lateness of the Straight Line

One of the most striking observations in the history of mathematics is how late the modern equation of the straight line actually appeared. Human beings constructed straight structures, measured distances, aligned stars, and built geometries for thousands of years before the symbolic compression

$$y = mx + c$$

became standard.

Ancient Egyptians and Babylonians solved practical linear problems. Euclid formalized geometric constructions and defined the properties of straight lines around 300 BC. Yet none of these systems represented lines algebraically through coordinate equations.

The major transition occurred with René Descartes in 1637 through *La Géométrie*, where geometry and algebra were unified via the Cartesian coordinate system. Coordinates allowed geometry to become representable through symbolic arithmetic. Yet even then, the modern slope-intercept form did not immediately appear.

The now familiar form

$$y = mx + c$$

only stabilized in the mid-nineteenth century through British mathematical texts such as those of Matthew O'Brien and George Salmon. This historical delay is deeply revealing. If the straight line were truly a self-evident Platonic object directly accessible to reason, its algebraic form should have appeared naturally and immediately. Instead, history suggests that symbolic compression emerges only after extensive operational and geometric stabilization. Finite Symbolic Mechanics interprets this historical trajectory differently:

> The straight line was never fundamentally an equation. The equation was merely the final symbolic compression of a finite operational process.

Geofinitism and the Foundations of Representation

Geofinitism begins from a simple commitment:

> All symbolic knowledge arises through finite measurement.

No symbol exists independently of a measurement process. Every representation inherits uncertainty, resolution limits, and provenance from the physical act of symbolic generation.

Within this framework:

- The external unknowable potential is denoted:

$$\sim continuum$$

This is not "the world" in a metaphysical sense. It is merely the generative potential from which measurements may arise.

- Measured symbolic reality is denoted:

$$\sim real$$

This refers only to that which has entered symbolic existence through finite measurement. Thus, Geofinitism rejects the assumption that mathematics directly accesses perfect external forms. Mathematics instead becomes a stabilized symbolic basin constructed through repeated finite interactions. The native operator of this framework is not equality.

It is approximation:

$$\sim$$

The tilde does not merely mean "approximately equal" in the classical sense. It denotes bounded measurable correspondence within a finite symbolic system.

Nexils and the Alphonic Limit

FSM introduces the concept of the *nexil*.

A nexil is the minimum finite geometric unit of symbolic representation obtainable through first-order measurement. It is not a dimensionless point. It is a finite uncertainty sphere. The minimum measurable interval associated with a nexil is called the Alphonic limit:

$$\delta$$

This represents the irreducible uncertainty of first-order measurement. One may think of it as the width of the smallest mark on a ruler, or the minimum geometric rounding interval obtainable without invoking higher-order endogenous modelling.

Thus every symbolic object carries uncertainty intrinsically. No perfect points exist. No exact coordinates exist. No perfect equality exists. Only bounded measurable symbolic stabilizations exist.

The Straight Line as a Geofinite Attractor

In classical mathematics, a straight line is often defined as a set of points satisfying an equation. FSM reverses this order completely. The straight line is instead understood as the simplest persistent symbolic propagation process. It is a geodesic attractor generated through repeated nexil alignment.

Let a nexil at step k be represented as:

$$N_k \sim (p_k, d_k)$$

where:

- p_k is the finite position sphere
- d_k is the local directional interaction primitive

The propagation rule becomes:

$$p_{k+1} \sim p_k + d_k \cdot \Delta$$

where:

- Δ is the finite propagation interval
- all quantities are bounded by uncertainty δ

The directional primitive remains stable:

$$d_{k+1} \sim d_k$$

This creates repeated collinear propagation. The straight line therefore emerges not as a static object but as a stabilized propagation basin.

Numerical Nexil Cascade

Consider the simplest non-trivial case:

$$m \sim 1$$

with:

$$\delta = 0.1$$

and propagation interval:

$$\Delta = 1$$

Initial nexil:

$$N_0 \sim (0, 0, (1, 1))$$

bounded within uncertainty interval:

$$[-0.05, +0.05]$$

The first propagation becomes:

$$x_1 \sim 1.0$$

$$y_1 \sim 1.0$$

Second propagation:

$$x_2 \sim 2.0$$

$$y_2 \sim 2.0$$

Third propagation:

$$x_3 \sim 3.0$$

$$y_3 \sim 3.0$$

The resulting symbolic structure forms a finite uncertainty tube around the classical ideal:

$$y \sim x$$

The line is therefore not infinitely precise. It is a stabilized finite symbolic corridor. Importantly, the uncertainty does not diverge chaotically because the propagation attractor constrains the symbolic trajectory.

This differs fundamentally from random noise accumulation.

The Emergence of Slope

Within FSM, slope is no longer a Platonic ratio.

Instead, slope becomes a local interaction coefficient governing nexil propagation.

Classically:

$$y = mx + c$$

In FSM this becomes:

$$y \sim mx + c$$

where:

- m is the local directional transmission coefficient
- c is the initial nexil offset from the chosen origin stabilization
- all quantities inherit alphonic uncertainty

Thus slope is operational rather than metaphysical.

It measures symbolic propagation tendency between neighboring nexils.

Matrices as Higher-Order Propagation Compressors

The reinterpretation of the straight line naturally extends into matrix mathematics.

Matrices are not fundamentally static symbolic arrays. They are compressed propagation operators. A matrix encodes repeated relational transformations between nexils. For example, a two-dimensional transformation matrix

$$A$$

acts not upon perfect vectors but upon finite uncertainty-bounded symbolic nexil chains. Eigenvectors therefore become stabilized propagation attractors. An eigenvector is simply a symbolic trajectory that re-enters its own geometric basin under repeated transformation. This interpretation explains why linear algebra appears so natural and powerful:

> It is fundamentally a symbolic compression framework for finite propagation dynamics.

Historical Reversal: Geometry Before Algebra

FSM predicts the historical order observed in mathematics. Ancient geometry preceded algebra because operational propagation preceded symbolic compression. Euclidean geometry already operated at the nexil level through straightedge and compass constructions. These were physical propagation systems constrained by finite measurement. The Cartesian coordinate system later compressed these constructions into symbolic lattices. Finally, nineteenth-century slope-intercept notation compressed the propagation itself into static algebraic shorthand. Thus history mirrors the FSM hierarchy:

1. Physical propagation

2. Geometric stabilization

3. Symbolic compression

4. Algebraic abstraction

Classical mathematics reverses this order conceptually.

FSM restores it.

Linear Mathematics and Quantum Mechanics

The implications become particularly interesting when viewed alongside quantum mechanics.

Quantum mechanics already struggles with measurement uncertainty and observational limits. FSM argues that these issues are not unique to quantum theory. They are foundational to all symbolic representation. The apparent precision of classical linear mathematics is therefore partly an artifact of symbolic compression. The suppression of uncertainty through perfect equality symbols creates the illusion of exactness. FSM restores uncertainty to the foundational layer itself. Thus:

- A point becomes an uncertainty sphere
- A line becomes an uncertainty tube
- A vector becomes a bounded directional propagation
- A matrix becomes a finite symbolic interaction field

This does not weaken mathematics. Rather, it grounds it operationally. Indeed, many paradoxes of modern physics may arise precisely because symbolic compressions are mistaken for ontological reality.

Discussion

The reinterpretation presented here does not reject classical mathematics. Rather, it situates classical mathematics within a broader operational and measurement-based framework. The classical equation

$$y = mx + c$$

remains enormously useful. But FSM argues that its usefulness emerges because it is a stable symbolic compression of finite propagation dynamics. This inversion has several implications:

Firstly, it reframes the foundations of linearity itself. Straightness is no longer absolute. It becomes the minimal-deviation propagation attractor permitted within the alphonic limit.

Secondly, it restores measurement uncertainty to the foundational layer of mathematics rather than treating uncertainty as a secondary engineering concern.

And thirdly, it provides a natural bridge between geometry, matrices, dynamical systems, and modern machine learning representations.

The propagation of nexils resembles temporal unfolding processes seen in matrices, embeddings, and nonlinear dynamical systems. Symbolic structures become stabilized trajectories rather than static abstract entities.

Fourth, it suggests that many historical mathematical developments were not discoveries of eternal truths, but progressive symbolic compressions of operational measurement procedures.

Finally, FSM provides a conceptual insert point between classical mathematics and Geofinitism.

Rather than rejecting the classical basin, FSM explains how the classical basin emerged historically and operationally through progressive symbolic stabilization. The straight line therefore becomes far more than a trivial object. It becomes the primitive geodesic of finite symbolic propagation itself.

Conclusion

Linear mathematics has long been regarded as the simplest layer of formal thought. Yet its history and operational structure reveal a far deeper story.

The modern equation of the straight line emerged only after centuries of geometric practice because the equation was never primary. The operational propagation process came first.

Finite Symbolic Mechanics reinterprets the straight line as a finite uncertainty-bounded symbolic attractor generated through repeated nexil propagation. The tilde operator replaces Platonic equality with bounded measurable correspondence. Matrices, vectors, and linear systems become higher-order compressions of finite symbolic flows.

In this view, mathematics is not detached from measurement but born from it. The line is no longer an infinite abstraction floating in Platonic space. It is the simplest stabilized trajectory that finite symbolic propagation can sustain within the bounds of measurable reality. And from this primitive geodesic, the vast edifice of linear mathematics unfolds.