The Attralucian Essays: Exploring the Finite

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The Attralucian Essays

Spherical Unity: A Geofinitist Lens

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Overview

This work extends the Geofinite reinterpretation of complex numbers into the domain of roots of unity, cyclic structures, and rotational closure geometries. Building upon earlier demonstrations that complex numbers may be understood as finite dynamical reconstruction operators rather than ontologically imaginary entities, we demonstrate that the so-called "missing roots" of polynomial equations arise naturally from projection collapse within scalar representations. (see essays ATT 24–25)

Within the framework of Finite Symbolic Mechanics (FSM), measured numbers are treated not as infinitesimal Platonic points but as finite geometric stabilizations possessing bounded admissibility and uncertainty. Under this reinterpretation, unity itself acquires a finite geometry, and the roots of unity emerge as admissible rotational closure states of reconstructed measurement manifolds.

The classical complex plane is reinterpreted as a minimal reconstruction surface that preserves rotational information lost under scalar projection. Euler's formula, primitive roots, Fourier decomposition, and cyclic algebraic structures are shown to arise naturally from finite rotational reconstruction geometry.

This work further argues that the historical development of complex numbers reflects the progressive stabilization of symbolic reconstruction operators required to preserve measurable relational invariants under finite observation.

section*Introduction

The history of complex numbers occupies an unusual place in mathematics. Emerging first as algebraic artifices in Renaissance polynomial theory, they gradually evolved into indispensable structures underlying modern analysis, signal theory, quantum mechanics, and engineering.

Yet the traditional interpretation remains philosophically unstable. Complex numbers are commonly introduced through the symbolic definition

$$i^2 = -1,$$

with the complex plane treated as a geometric extension of the real line into an orthogonal imaginary dimension.

Within the Geofinite framework and Finite Symbolic Mechanics (FSM), this interpretation is reconsidered at a fundamental level. The present work argues that complex numbers do not represent metaphysical extensions beyond measured reality. Instead, they arise naturally as stable symbolic compressions of rotational geometry reconstructed from finite measurement and relational delay.

This reinterpretation becomes especially powerful when applied to one of the deepest structures in mathematics:

$$x^n = 1,$$

the roots of unity. Traditionally viewed as points distributed around the complex unit circle, the roots are here reinterpreted as finite rotational closure states emerging from the projection and reconstruction of finite geometric unity itself.

section*From Platonic Numbers to Finite Geometry

Measured Numbers

Within classical mathematics, numbers are treated as exact scalar points possessing infinite precision. FSM rejects this assumption on operational grounds.

Every physical measurement possesses finite resolution, bounded uncertainty, admissibility constraints, and finite symbolic representation. Consequently, a measured number cannot correspond to an infinitesimal point. Instead, measured quantities occupy finite geometric regions within admissible measurement space.

Unity Reinstantiated

The classical number 1 is therefore reinterpreted as a finite geometric stabilization:

$$1 \sim \mathcal{S}_1,$$

where $\mathcal{S}_1$ denotes a bounded finite geometry associated with unity. In the absence of privileged orientation, this stabilization naturally tends toward spherical geometry.

section*Projection and Reconstruction

Projection Collapse

Scalar representations preserve only partial geometric information. Consider rotational motion projected onto a single axis. Distinct rotational states may map onto identical scalar values:

$$x = \cos(\theta).$$

The projection preserves magnitude while destroying orientation information.

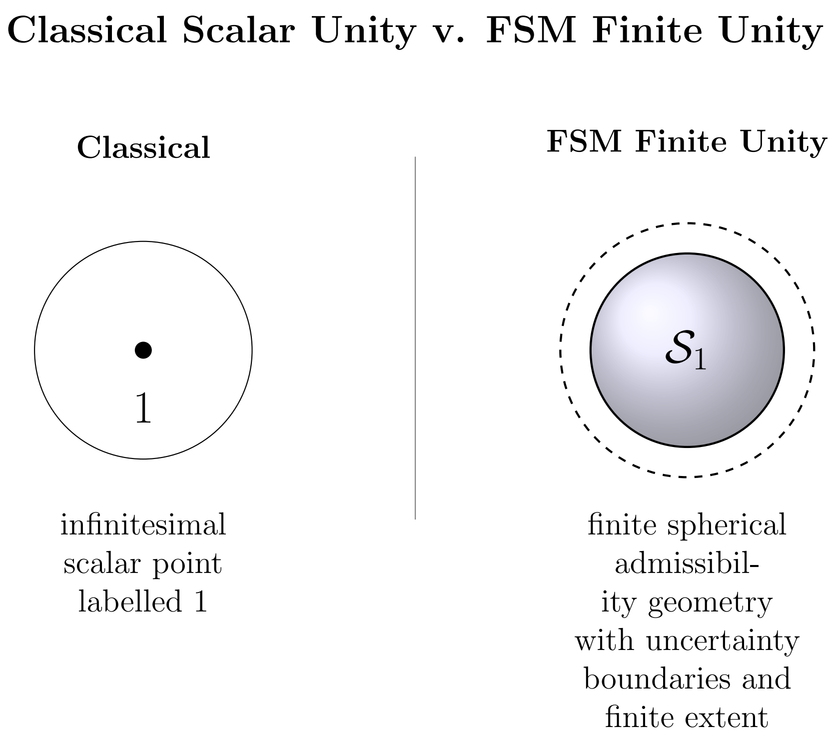

Figure 1: Within FSM, unity is treated as a finite admissible geometric stabilization rather than an infinitesimal scalar point.

Projection Collapse of Rotational Geometry

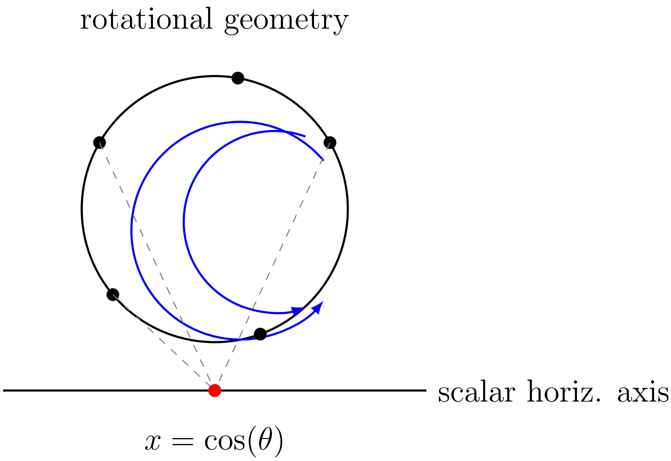

Figure 2: Scalar projection destroys rotational orientation information while preserving magnitude relations.

Delay Reconstruction

Let $x(t)$ represent a measured signal. Construct the delay embedding:

$$X(t) = (x(t), x(t - \tau)).$$

Rotational geometries emerge naturally within the reconstructed space.

section*Complex Numbers as Reconstruction Operators

The Imaginary Unit Reinterpreted

Within FSM, the imaginary unit no longer represents an ontological imaginary dimension. Instead, i acts as a symbolic compression of rotational displacement within reconstructed phase geometry.

Euler's formula

$$e^{i\theta} = \cos(\theta) + i \sin(\theta)$$

becomes compressed notation for rotational traversal around the reconstructed finite geometry.

Quarter Rotation Geometry

The identity $i^2 = -1$ corresponds geometrically to two successive quarter-rotations producing inversion. Likewise, $i^4 = 1$ represents closure after four quarter-rotational transformations.

section*Roots of Unity as Rotational Closure States

The Roots Problem Reconsidered

Classically, the equation $x^n = 1$ possesses n roots. Within scalar projection, however, many roots appear "missing." FSM reframes the issue: the roots are not absent entities existing within an imaginary domain. They are rotational states collapsed by scalar projection.

Cube Roots of Unity

The cube roots correspond to three rotational closure states:

$$0, \quad \frac{2\pi}{3}, \quad \frac{4\pi}{3}.$$

Their scalar projections collapse partially onto identical real values.

Primitive Roots

Primitive roots correspond to minimal rotational phase steps that traverse the full cyclic closure geometry before returning to unity. Non-primitive roots represent inherited lower-order subcycles.

section*Fourier Structure and Phase Alphabets

Rotational Basis Functions

The Discrete Fourier Transform employs

$$e^{-2\pi ikn/N}$$

as rotational basis functions. Within FSM, these become finite cyclic reconstruction operators preserving phase relations across sampled measurements.

Phase Alphabets

The roots of unity form discrete cyclic alphabets of rotational phase states. Fourier decomposition thus becomes reconstruction through weighted rotational phase superposition.

section*The Historical Unfolding of Reconstruction Geometry

The historical development of complex numbers may be reinterpreted as the progressive stabilization of reconstruction operators required to preserve finite relational geometry under projection constraints. This unfolding proceeds from classical scalar numbers, through the recognition of projection insufficiency, the introduction of complex numbers, the emergence of rotational geometry, the development of delay reconstruction (including Takens embedding), and finally to the full recognition of finite relational geometry within the FSM framework.

section*The Unity Sphere and the Reconstruction Plane

The complex plane is not primary; the unity sphere is primary. The plane emerges as a reconstructed projection manifold that preserves rotational structure lost under scalar collapse. The sphere has rotational symmetry in three dimensions; the complex plane captures only the equatorial slice that preserves orientation information.

It's of note the Alphonic Limit – the boundary of first-order measurement – is spherical under isotropic uncertainty. However, the Nexil itself is not assumed to be spherical. Its geometry is modelled, not known. The 'unity sphere' used here is a reconstruction device, not a claim of a 'thing', it is instantiated as an endogenous symbol via an endogenous *Generonic Process*.

section*Conclusion

The reinterpretation developed throughout this work replaces the traditional metaphysical framing of complex numbers with a finite geometric reconstruction framework grounded in measurement, delay, and relational structure.

The roots of unity are no longer understood as mysterious solutions residing within an abstract imaginary domain. They become admissible rotational closure states arising naturally from finite geometric unity under projection and reconstruction. The imaginary unit itself is re-situated: not as an ontological extension, but as a symbolic compression of rotational-delay geometry. The historical success of complex numbers is therefore neither accidental nor mystical. Complex arithmetic persists because it preserves invariant rotational structures emerging from finite measured dynamics. The complex plane survives because it is an extraordinarily stable reconstruction geometry.

In this sense, mathematics itself appears not as a catalogue of eternal Platonic objects, but as an evolving symbolic dynamical system stabilizing around representations that preserve measurable relational invariants under finite observation.

References

Referenced Essays Attralucians 24 and 26:

ATT 24 complex numbers dynamical reconstruction
ATT 25 complex analysis takens embedding.pdf

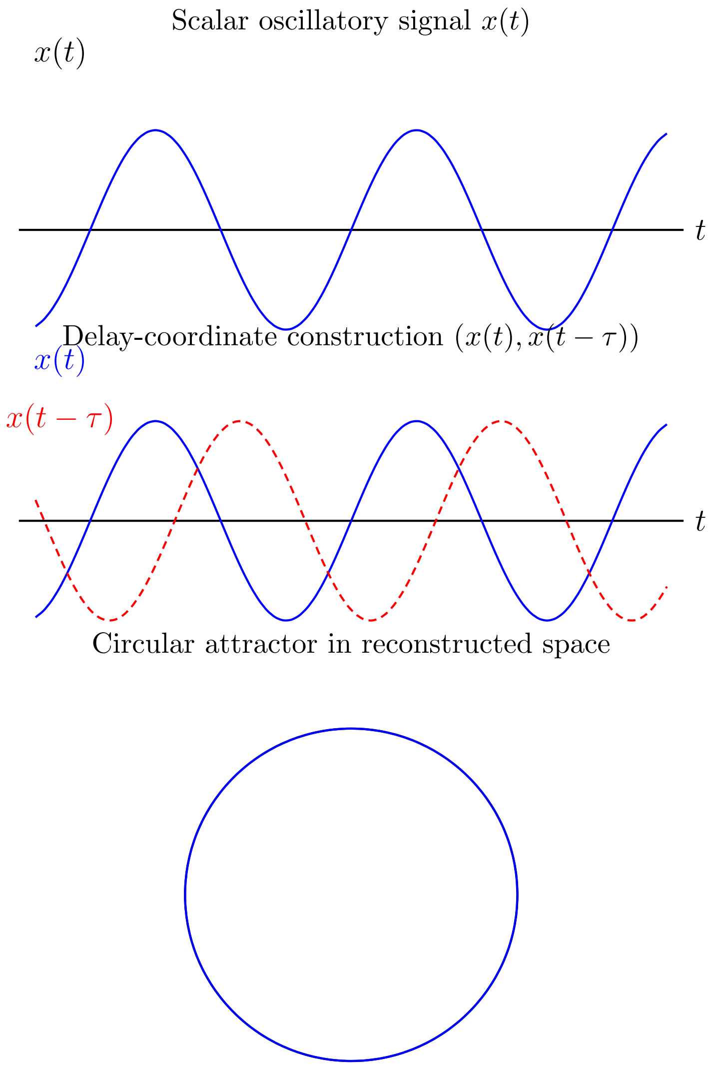

Figure 3: Rotational geometry emerges naturally through finite delay reconstruction.

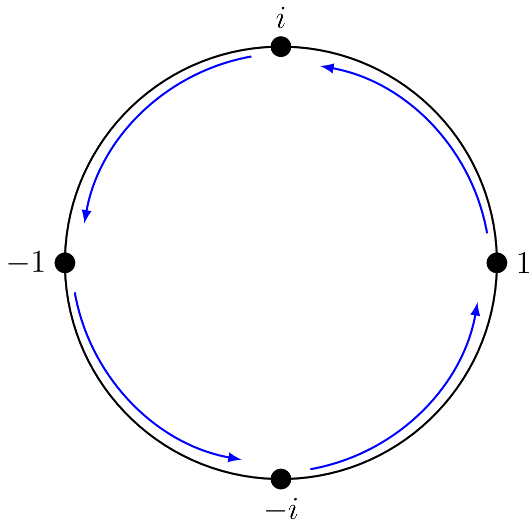

Figure 4: The imaginary unit acts as a rotational-delay operator within reconstructed phase geometry.

Cube Roots as Rotational Closure States

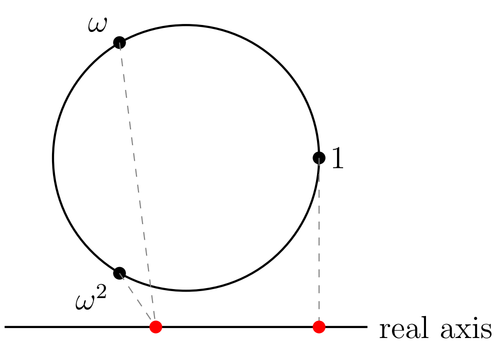

Figure 5: The "missing" roots are unresolved rotational states collapsed by projection.

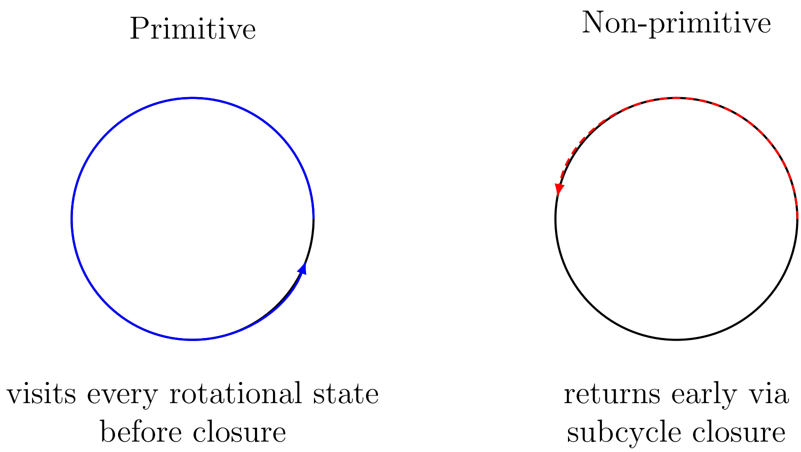

Figure 6: Primitive roots preserve maximal cyclic resolution.

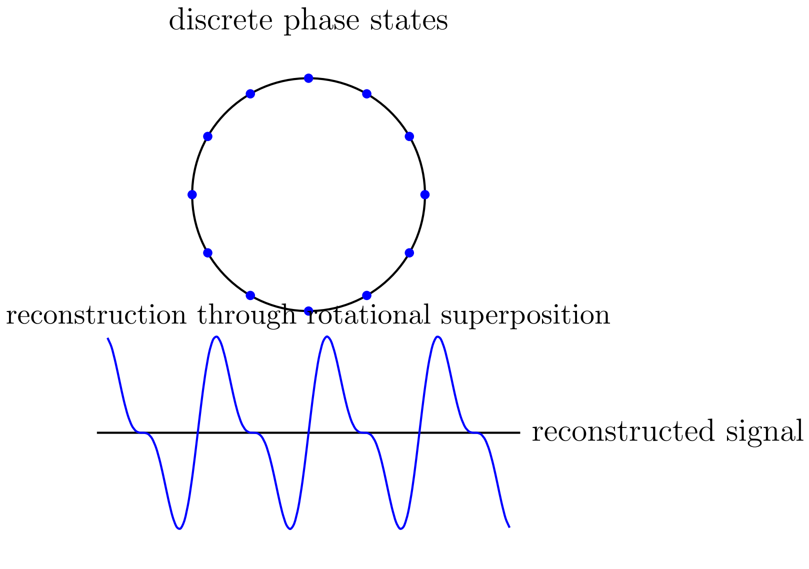

Figure 7: The Fourier basis acts as a finite cyclic phase alphabet for signal reconstruction.

Figure Placeholder 8: FSM Historical Narrative Flow

Classical Scalar Numbers

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Projection Insufficiency

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Complex Numbers Introduced

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Rotational Geometry Emerges

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Delay Reconstruction

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Takens Embedding

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Finite Relational Geometry

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FSM Reinterpretation

Figure 8: The historical emergence of complex numbers may be understood as progressive reconstruction stabilization.

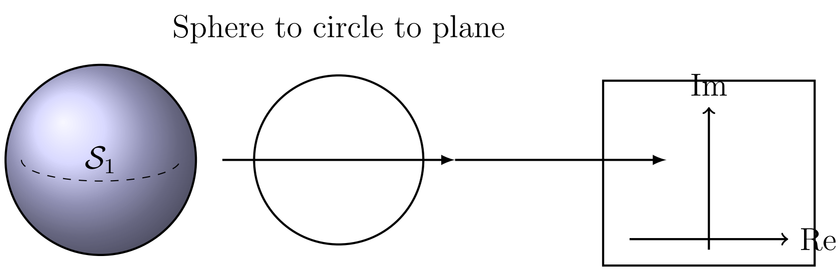

Figure 9: The complex plane emerges as a reconstruction slice of finite geometric unity. Sphere to circle to plane